Zadatak 11. Izračunaj $\cos(\alpha+\beta)$ i $\cos(\alpha-\beta)$ ako je $\sin\left(\frac{\pi}{2}-\alpha\right)=0.8$, $\sin\left(\frac{\pi}{2}-\beta\right)=\frac{15}{17}$, $0<\alpha<\frac{\pi}{2}$, $\frac{3\pi}{2}<\beta<2\pi$.

Rješenje. $0<\alpha<\frac{\pi}{2} \implies \cos\alpha,\ \sin\alpha,\ \operatorname{tg}\alpha,\ \operatorname{ctg}\alpha>0$

$\frac{3\pi}{2}<\beta<2\pi \implies \cos\beta>0,\ \sin\beta,\ \operatorname{tg}\beta,\ \operatorname{ctg}\beta<0$

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\underbrace{\sin\frac{\pi}{2}}_{=1}\cos\alpha-\underbrace{\cos\frac{\pi}{2}}_{=0}\sin\alpha=\frac{8}{10}=\frac{4}{5} \implies \cos\alpha=\frac{4}{5}$$

$$\sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

$$\sin\left(\frac{\pi}{2}-\beta\right)=\underbrace{\sin\frac{\pi}{2}}_{=1}\cos\beta-\underbrace{\cos\frac{\pi}{2}}_{=0}\sin\beta=\frac{15}{17} \implies \cos\beta=\frac{15}{17}$$

$$\sin\beta=-\sqrt{1-\cos^2\beta}=-\sqrt{1-\frac{225}{289}}=-\frac{8}{17}$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{4}{5}\cdot\frac{15}{17}-\frac{3}{5}\cdot\left(-\frac{8}{17}\right)=\frac{60+24}{85}=\frac{84}{85};$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{4}{5}\cdot\frac{15}{17}+\frac{3}{5}\cdot\left(-\frac{8}{17}\right)=\frac{60-24}{85}=\frac{36}{85}$$