

Zadatak 16. Ako je $\alpha + \beta = \frac{3\pi}{4}$; $\cos \beta = \frac{3\sqrt{7}}{8}$, $\frac{\pi}{2} < \alpha < \pi$, koliko je $\sin \alpha$?

Rješenje.

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{63}{64}} = \frac{1}{8}$$

$$\sin(\alpha + \beta) = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos(\alpha + \beta) = \cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{\sqrt{2}}{2}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = -\frac{\sqrt{2}}{2}$$

$$\sin \alpha \cdot \frac{3\sqrt{7}}{8} + \frac{1}{8} \cdot \cos \alpha = \frac{\sqrt{2}}{2} \quad / \cdot 8$$

$$\cos \alpha \cdot \frac{3\sqrt{7}}{8} - \frac{1}{8} \cdot \sin \alpha = -\frac{\sqrt{2}}{2} \quad / \cdot 8$$

$$3\sqrt{7} \sin \alpha + \cos \alpha = 4\sqrt{2} \quad / \cdot (-3\sqrt{7})$$

$$-\sin \alpha + 3\sqrt{7} \cos \alpha = -4\sqrt{2}$$

$$\left. \begin{array}{l} -63 \sin \alpha - 3\sqrt{7} \cos \alpha = -21\sqrt{14} \\ -\sin \alpha + 3\sqrt{7} \cos \alpha = -4\sqrt{2} \end{array} \right\} +$$

$$-64 \sin \alpha = -4\sqrt{2}(1 + 3\sqrt{7}) \quad / : (-64)$$

$$\sin \alpha = \frac{-4\sqrt{2}(1 + 3\sqrt{7})}{-64} = \frac{\sqrt{2}(1 + 3\sqrt{7})}{16}$$