

Zadatak 19. Ako je $\sin x = \frac{1}{\sqrt{5}}$, $\sin y = \frac{1}{\sqrt{10}}$, te $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, onda je $x + y = \frac{\pi}{4}$. Dokaži!

Rješenje. Dovoljno je vidjeti $\cos(x + y) = \frac{\sqrt{2}}{2}$.

$$\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \frac{1}{5}} = \frac{2}{\sqrt{5}}$$

$$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - \frac{1}{10}} = \frac{3}{\sqrt{10}}$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y = \frac{2}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} - \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}} = \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow x + y = \frac{\pi}{4}.$$