Zadatak 26. Ako su α i β šiljasti kutovi i $\operatorname{tg}\alpha = \frac{a\sqrt{3}}{4-a}$, $\operatorname{tg}\beta = \frac{a-1}{\sqrt{3}}$, $a \in \langle 1, 4\rangle$, tada je $\alpha - \beta = 30^\circ$. Dokaži!

Rješenje.

$$\operatorname{tg}(\alpha-\beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\beta} = \frac{\frac{a\sqrt{3}}{4-a} - \frac{a-1}{\sqrt{3}}}{1+\frac{a\sqrt{3}}{4-a}\cdot\frac{a-1}{\sqrt{3}}} = \frac{\frac{3a-(a-1)(4-a)}{\sqrt{3}(4-a)}}{1+\frac{a(a-1)}{4-a}}$$

$$= \frac{\frac{3a-4a+a^2+4-a}{\sqrt{3}(4-a)}}{\frac{4-a+a^2-a}{4-a}} = \frac{a^2-2a+4}{\sqrt{3}(a^2-2a+4)} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\Longrightarrow \quad \alpha - \beta = 30^\circ.$$

Provjeri da je $\operatorname{tg}(\alpha-\beta) = \frac{\sqrt{3}}{3}$.