Zadatak 35. Dokaži sljedeće identitete:

1) $\sin(x-y)\cdot\sin(x+y)=\cos^2 y-\cos^2 x$;

2) $\cos(x+y)\cdot\cos(x-y)=\cos^2 y-\sin^2 x$;

3) $\cos(x+y)\cdot\cos(x-y)+\sin(x+y)\cdot\sin(x-y)$
$=\cos^2 y-\sin^2 y$;

4) $\dfrac{\sin(x+y)}{\cos x\cdot\cos y}=\operatorname{tg} x+\operatorname{tg} y$;

5) $\dfrac{\sin(x-y)}{\operatorname{tg} x-\operatorname{tg} y}=\cos x\cdot\cos y$;

6) $\dfrac{\cos(x+y)\cdot\cos(x-y)}{\sin^2 x\cdot\sin^2 y}=\operatorname{ctg}^2 x\cdot\operatorname{ctg}^2 y-1$;

7) $\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}=\dfrac{\operatorname{tg}\alpha+\operatorname{ctg}\beta}{\operatorname{ctg}\beta-\operatorname{tg}\alpha}$;

8) $\dfrac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)}=\operatorname{tg} x\cdot\operatorname{ctg} y$.

Rješenje. **1)**

$$\begin{aligned}
\sin(x-y)\cdot\sin(x+y)&=\cos^2 y-\cos^2 x\\
(\sin x\cos y-\cos x\sin y)\cdot(\sin x\cos y+\cos x\sin y)&=\cos^2 y-\cos^2 x\\
\sin^2 x\cos^2 y-\cos^2 x\sin^2 y&=\cos^2 y-\cos^2 x\\
(1-\cos^2 x)\cos^2 y-\cos^2 x(1-\cos^2 y)&=\cos^2 y-\cos^2 x\\
\cos^2 y-\cos^2 x\cos^2 y-\cos^2 x+\cos^2 x\cos^2 y&=\cos^2 y-\cos^2 x\\
\cos^2 y-\cos^2 x&=\cos^2 y-\cos^2 x
\end{aligned}$$

2)

$$\begin{aligned}
\cos(x+y)\cdot\cos(x-y)&=\cos^2 y-\sin^2 x\\
(\cos x\cos y-\sin x\sin y)\cdot(\cos x\cos y+\sin x\sin y)&=\cos^2 y-\sin^2 x\\
\cos^2 x\cos^2 y-\sin^2 x\sin^2 y&=\cos^2 y-\sin^2 x\\
(1-\sin^2 x)\cos^2 y-\sin^2 x(1-\cos^2 y)&=\cos^2 y-\sin^2 x\\
\cos^2 y-\sin^2 x\cos^2 y-\sin^2 x+\sin^2 x\cos^2 y&=\cos^2 y-\sin^2 x\\
\cos^2 y-\sin^2 x&=\cos^2 y-\sin^2 x
\end{aligned}$$

3)

$$\begin{aligned}
\underbrace{\cos(x+y)\cdot\cos(x-y)}_{=\cos^2 y-\sin^2 x\ (\text{pod } \mathbf{2)})}+\underbrace{\sin(x+y)\cdot\sin(x-y)}_{=\cos^2 y-\cos^2 x\ (\text{pod } \mathbf{1)})}&=\cos^2 y-\sin^2 y\\
\cos^2 y-\sin^2 x+\cos^2 y-\cos^2 x&=\cos^2 y-\sin^2 y\\
\cos^2 y+\cos^2 y-(\sin^2 x+\cos^2 x)&=\cos^2 y-\sin^2 y\\
\cos^2 y+(1-\sin^2 y)-1&=\cos^2 y-\sin^2 y\\
\cos^2 y+1-\sin^2 y-1&=\cos^2 y-\sin^2 y\\
\cos^2 y-\sin^2 y&=\cos^2 y-\sin^2 y
\end{aligned}$$

4)

$$\begin{aligned}
\frac{\sin(x+y)}{\cos x \cdot \cos y} &= \operatorname{tg} x + \operatorname{tg} y \\
\frac{\sin x \cdot \cos y + \cos x \cdot \sin y}{\cos x \cdot \cos y} &= \operatorname{tg} x + \operatorname{tg} y \\
\frac{\sin x \cdot \cos y}{\cos x \cdot \cos y} + \frac{\cos x \cdot \sin y}{\cos x \cdot \cos y} &= \operatorname{tg} x + \operatorname{tg} y \\
\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} &= \operatorname{tg} x + \operatorname{tg} y \\
\operatorname{tg} x + \operatorname{tg} y &= \operatorname{tg} x + \operatorname{tg} y
\end{aligned}$$

5)

$$\begin{aligned}
\frac{\sin(x-y)}{\operatorname{tg} x - \operatorname{tg} y} &= \cos x \cdot \cos y \\
\frac{\sin x \cos y - \cos x \sin y}{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}} &= \cos x \cdot \cos y \\
\frac{\sin x \cos y - \cos x \sin y}{\frac{\sin x \cos y - \cos x \sin y}{\cos x \cdot \cos y}} &= \cos x \cdot \cos y \\
\cos x \cdot \cos y &= \cos x \cdot \cos y
\end{aligned}$$

6)

$$\begin{aligned}
\frac{\cos(x+y) \cdot \cos(x-y)}{\sin^2 x \cdot \sin^2 y} &= \operatorname{ctg}^2 x \cdot \operatorname{ctg}^2 y - 1 \\
\frac{(\cos x \cdot \cos y - \sin x \cdot \sin y) \cdot (\cos x \cdot \cos y + \sin x \cdot \sin y)}{\sin^2 x \cdot \sin^2 y} &= \operatorname{ctg}^2 x \cdot \operatorname{ctg}^2 y - 1 \\
\frac{\cos^2 x \cdot \cos^2 y - \sin^2 x \cdot \sin^2 y}{\sin^2 x \cdot \sin^2 y} &= \operatorname{ctg}^2 x \cdot \operatorname{ctg}^2 y - 1 \\
\frac{\cos^2 x \cdot \cos^2 y}{\sin^2 x \cdot \sin^2 y} - \frac{\sin^2 x \cdot \sin^2 y}{\sin^2 x \cdot \sin^2 y} &= \operatorname{ctg}^2 x \cdot \operatorname{ctg}^2 y - 1 \\
\operatorname{ctg}^2 x \cdot \operatorname{ctg}^2 y - 1 &= \operatorname{ctg}^2 x \cdot \operatorname{ctg}^2 y - 1
\end{aligned}$$

7)

$$\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}=\frac{\operatorname{tg}\alpha+\operatorname{ctg}\beta}{\operatorname{ctg}\beta-\operatorname{tg}\alpha}$$

$$\frac{\cos\alpha\cdot\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cdot\cos\beta-\sin\alpha\sin\beta}=\frac{\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\cos\beta}{\sin\beta}}{\dfrac{\cos\beta}{\sin\beta}-\dfrac{\sin\alpha}{\cos\alpha}}$$

$$\frac{\cos\alpha\cdot\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cdot\cos\beta-\sin\alpha\sin\beta}=\frac{\dfrac{\cos\alpha\cdot\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cdot\sin\beta}}{\dfrac{\cos\alpha\cdot\cos\beta-\sin\alpha\sin\beta}{\sin\beta\cdot\cos\alpha}}$$

$$\frac{\cos\alpha\cdot\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cdot\cos\beta-\sin\alpha\sin\beta}=\frac{\cos\alpha\cdot\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cdot\cos\beta-\sin\alpha\sin\beta}$$

8)

$$\frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)}=\operatorname{tg}x\cdot\operatorname{ctg}y$$

$$\frac{\sin x\cos y+\cos x\sin y+\sin x\cos y-\cos x\sin y}{\sin x\cos y+\cos x\sin y-\sin x\cos y+\cos x\sin y}=\operatorname{tg}x\cdot\operatorname{ctg}y$$

$$\frac{2\sin x\cos y}{2\cos x\sin y}=\operatorname{tg}x\cdot\operatorname{ctg}y$$

$$\operatorname{tg}x\cdot\operatorname{ctg}y=\operatorname{tg}x\cdot\operatorname{ctg}y$$