Zadatak 9. Ako je $\sin x = \frac{1}{\sqrt{3}}$, $\frac{\pi}{2} < x < \pi$, izračunaj $\cos 2x$ i $\operatorname{tg}\left(2x - \frac{\pi}{4}\right)$.

Rješenje.

$$\frac{\pi}{2} < x < \pi \implies \cos x < 0,$$

$$\pi < 2x < 2\pi \implies \sin 2x < 0,$$

$$\cos x = -\sqrt{1-\sin^2 x} = -\sqrt{1-\frac{1}{3}} = -\sqrt{\frac{2}{3}}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\sin 2x = -\sqrt{1-\cos^2 2x} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

$$\operatorname{tg} 2x = \frac{\sin 2x}{\cos 2x} = \frac{-\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = -2\sqrt{2}$$

$$\operatorname{tg}\left(2x - \frac{\pi}{4}\right) = \frac{\operatorname{tg} 2x - \operatorname{tg}\frac{\pi}{4}}{1 + \operatorname{tg} 2x \cdot \operatorname{tg}\frac{\pi}{4}} = \frac{-2\sqrt{2}-1}{1-2\sqrt{2}\cdot 1} = \frac{-2\sqrt{2}-1}{1-2\sqrt{2}} = \frac{2\sqrt{2}+1}{2\sqrt{2}-1} \approx 2.0938$$