

Zadatak 11. Ako je $\sin x = \frac{9}{41}$, $\cos y = \frac{60}{61}$, te $x \in \langle 0, \frac{\pi}{2} \rangle$, $y \in \langle 0, \frac{\pi}{2} \rangle$, koliko je $\sin^2 \frac{x-y}{2}$?

Rješenje. $x, y \in \langle 0, \frac{\pi}{2} \rangle \implies \cos x, \sin y > 0$

$$\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \frac{81}{1681}} = \frac{40}{41}$$

$$\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - \frac{3600}{3721}} = \frac{11}{61}$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y = \frac{40}{41} \cdot \frac{60}{61} + \frac{9}{41} \cdot \frac{11}{61} = \frac{2499}{2501}$$

$$\sin \frac{x-y}{2} = \sqrt{\frac{1 - \cos(x-y)}{2}} = \sqrt{\frac{1 - \frac{2499}{2501}}{2}} = \sqrt{\frac{1}{2501}} = \frac{1}{\sqrt{2501}}$$

$$\sin^2 \frac{x-y}{2} = \frac{1}{2501}.$$