Zadatak 12. Izračunaj:

1) $\cos\frac{\alpha}{2}$, ako je $\operatorname{ctg}\alpha = -\frac{7}{24}$, $\frac{5\pi}{2} < \alpha < 3\pi$;

2) $\sin\alpha$, ako je $\cos 2\alpha = \frac{7}{8}$, $\frac{3\pi}{4} < \alpha < \pi$;

3) $\operatorname{ctg}\frac{\alpha}{2}$, ako je $\cos 2\alpha = -\frac{527}{625}$, $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$;

4) $\operatorname{tg}\frac{\alpha}{2}$, ako je $\operatorname{ctg}\alpha = -2.4$, $\frac{3\pi}{2} < \alpha < 2\pi$.

Rješenje. **1)** $\frac{5\pi}{2} < \alpha < 3\pi \implies \frac{\pi}{2} < \alpha < \pi \implies \sin\alpha > 0,\ \cos\alpha < 0$;
$\frac{\pi}{2} < \alpha < \pi \implies \frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2} \implies \cos\frac{\alpha}{2} > 0$:

$$\sin\alpha = \sqrt{\frac{1}{1+\operatorname{ctg}^2\alpha}} = \sqrt{\frac{1}{1+\frac{49}{576}}} = \sqrt{\frac{1}{\frac{625}{576}}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{576}{625}} = -\sqrt{\frac{49}{625}} = -\frac{7}{25}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1-\frac{7}{25}}{2}} = \sqrt{\frac{\frac{18}{25}}{2}} = \sqrt{\frac{9}{25}} = \frac{3}{5};$$

2) $\frac{3\pi}{4} < \alpha < \pi \implies \sin\alpha > 0$:

$$\sin\alpha = \sqrt{\frac{1-\cos 2\alpha}{2}} = \sqrt{\frac{1-\frac{7}{8}}{2}} = \sqrt{\frac{\frac{1}{8}}{2}} = \sqrt{\frac{1}{16}} = \frac{1}{4};$$

3) $\frac{\pi}{4} < \alpha < \frac{\pi}{2} \implies \sin\alpha,\ \cos\alpha > 0$:

$$\sin\alpha = \sqrt{\frac{1-\cos 2\alpha}{2}} = \sqrt{\frac{1+\frac{527}{625}}{2}} = \sqrt{\frac{\frac{1152}{625}}{2}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

$$\cos\alpha = \sqrt{\frac{1+\cos 2\alpha}{2}} = \sqrt{\frac{1-\frac{527}{625}}{2}} = \sqrt{\frac{\frac{98}{625}}{2}} = \sqrt{\frac{49}{625}} = \frac{7}{25}$$

$$\operatorname{ctg}\frac{\alpha}{2} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{1+\frac{7}{25}}{\frac{24}{25}} = \frac{32}{24} = \frac{4}{3};$$

4) $\frac{3\pi}{2} < \alpha < 2\pi \implies \frac{3\pi}{4} < \frac{\alpha}{2} < \pi \implies \sin\frac{\alpha}{2} > 0,\ \cos\frac{\alpha}{2},\ \operatorname{tg}\frac{\alpha}{2} < 0:$

$$\operatorname{tg}\alpha = \frac{1}{\operatorname{ctg}\alpha} = -\frac{1}{2.4} = -\frac{5}{12}$$

$$-\frac{5}{12} = \frac{2\operatorname{tg}\frac{\alpha}{2}}{1-\operatorname{tg}^2\frac{\alpha}{2}}$$

$$-5\left(1-\operatorname{tg}^2\frac{\alpha}{2}\right) = 12\cdot\left(2\operatorname{tg}\frac{\alpha}{2}\right)$$

$$5\operatorname{tg}^2\frac{\alpha}{2} - 5 = 24\operatorname{tg}\frac{\alpha}{2}$$

$$5\operatorname{tg}^2\frac{\alpha}{2} - 24\operatorname{tg}\frac{\alpha}{2} - 5 = 0 \quad \left(\operatorname{tg}\frac{\alpha}{2} = t\right)$$

$$5t^2 - 24t - 5 = 0$$

$$t_{1,2} = \frac{24 \pm \sqrt{576+4\cdot5\cdot5}}{10} = \frac{24\pm\sqrt{576+100}}{10} = \frac{24\pm 26}{10} = \frac{12\pm13}{5}$$

$t_1 = -\frac{1}{5}, \quad t_2 = 5;$

Kako mora vrijediti $\operatorname{tg}\frac{\alpha}{2} < 0$ sljedi:

$$\operatorname{tg}\frac{\alpha}{2} = -\frac{1}{5}.$$