

Zadatak 20. Ako je $\cos \frac{x}{2} - \sin \frac{x}{2} = \frac{1}{5}$, $0 < x < \frac{\pi}{2}$, koliko je $\operatorname{ctg} 2x$?

Rješenje. $0 < x < \frac{\pi}{2} \implies \cos x > 0$

$$\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2 = \cos^2 \frac{x}{2} - 2 \cos \frac{x}{2} \sin \frac{x}{2} + \sin^2 \frac{x}{2} = 1 - \sin x = \frac{1}{25} \implies \sin x = \frac{24}{25}$$

$$\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \frac{576}{625}} = \frac{7}{25}$$

$$\operatorname{ctg} x = \frac{\cos x}{\sin x} = \frac{\frac{7}{25}}{\frac{24}{25}} = \frac{7}{24}$$

$$\operatorname{ctg} 2x = \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x} = \frac{\frac{49}{576} - 1}{2 \cdot \frac{7}{24}} = -\frac{527}{336}$$