

**Zadatak 25.** Ako je  $\cos 2\alpha = m$ ,  $|m| \leq 1$ , koliko je:

1)  $\sin^6 \alpha + \cos^6 \alpha$ ;

2)  $\sin^6 \alpha - \cos^6 \alpha$ ?

**Rješenje.**

1)

$$\begin{aligned}\sin^6 \alpha + \cos^6 \alpha &= (\sin^2 \alpha)^3 + (\cos^2 \alpha)^3 = (\sin^2 \alpha + \cos^2 \alpha)(\cos^4 \alpha - \cos^2 \alpha \sin^2 \alpha + \sin^4 \alpha) \\ &= (\cos^2 \alpha - \sin^2 \alpha)^2 + \cos^2 \alpha \sin^2 \alpha = \cos^2 2\alpha + \frac{1}{4} \sin^2 2\alpha \\ &= \cos^2 2\alpha + \frac{1}{4}(1 - \cos^2 2\alpha) = m^2 + \frac{1}{4}(1 - m^2) = \frac{3}{4}m^2 + \frac{1}{4} = \frac{1 + 3m^2}{4}\end{aligned}$$

2)

$$\begin{aligned}\sin^6 \alpha - \cos^6 \alpha &= (\sin^2 \alpha)^3 - (\cos^2 \alpha)^3 = (\sin^2 \alpha - \cos^2 \alpha)(\cos^4 \alpha + \cos^2 \alpha \sin^2 \alpha + \sin^4 \alpha) \\ &= -\cos 2\alpha(\sin^2 \alpha + \cos^2 \alpha)^2 - \cos^2 \alpha \sin^2 \alpha = -\cos 2\alpha\left(1 - \frac{1}{4} \sin^2 2\alpha\right) \\ &= -\cos 2\alpha\left[1 - \frac{1}{4}(1 - \cos^2 2\alpha)\right] = -\cos 2\alpha\left(\frac{3}{4} + \frac{1}{4} \cos^2 2\alpha\right) \\ &= -m\left(\frac{3}{4} + \frac{1}{4}m^2\right) = -m\frac{m^2 + 3}{4}\end{aligned}$$