Zadatak 26. Ako je $\cos 4\alpha = \frac{1}{3}$, koliko je $\sin^6 \alpha + \cos^6 \alpha$?

Rješenje.

$$\begin{aligned}
\sin^6 \alpha + \cos^6 \alpha &= (\sin^2 \alpha)^3 + (\cos^2 \alpha)^3 = (\sin^2 \alpha + \cos^2 \alpha)(\cos^4 \alpha - \cos^2 \alpha \sin^2 \alpha + \sin^4 \alpha) \\
&= (\sin^2 \alpha - \cos^2 \alpha)^2 + \sin^2 \alpha \cos^2 \alpha = \cos^2 2\alpha + \frac{1}{4} \sin^2 2\alpha \\
&= \cos^2 2\alpha + \frac{1}{4} - \frac{1}{4} \cos^2 2\alpha = \frac{1}{4} + \frac{3}{4} \cos^2 2\alpha = \frac{1}{4} + \frac{3}{4} \cdot \frac{1 + \cos 4\alpha}{2} \\
&= \frac{1}{4} + \frac{3}{4} \cdot \frac{1 + \frac{1}{3}}{2} = \frac{1}{4} + \frac{3}{4} \cdot \frac{4}{6} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}
\end{aligned}$$