**Zadatak 27.** Odredi najmanju vrijednost funkcije

$$f(x)=\frac{\operatorname{ctg}2x-\operatorname{tg}2x}{1+\sin\left(\frac{5\pi}{2}-8x\right)}$$

na intervalu $\left\langle 0,\frac{\pi}{4}\right\rangle$ .

***Rješenje.***

$$f(x)=\frac{\operatorname{ctg}2x-\operatorname{tg}2x}{1+\sin\left(\frac{5\pi}{2}-8x\right)}=\frac{\frac{1}{\operatorname{tg}2x}-\operatorname{tg}2x}{1+\underbrace{\sin\frac{5\pi}{2}}_{=\sin\frac{\pi}{2}=1}\cos 8x-\underbrace{\cos\frac{5\pi}{2}}_{=\cos\frac{\pi}{2}=0}\sin 8x}=\frac{\frac{1-\operatorname{tg}^2 2x}{\operatorname{tg}2x}}{1+\cos 8x}$$

$$=\frac{1-\operatorname{tg}^2 2x}{\operatorname{tg}2x(1+\cos 8x)}=\frac{\frac{\cos^2 2x-\sin^2 2x}{\cos^2 2x}}{\frac{\sin 2x}{\cos 2x}(1+\cos 8x)}=\frac{\cos 4x}{\sin 2x\cos 2x(1+\cos 8x)}$$

$$=\frac{\cos 4x}{\frac{1}{2}\sin 4x(1+\cos 8x)}=\frac{2\cos 4x}{\sin 4x\cdot 2\cos^2 4x}=\frac{1}{\frac{1}{2}\sin 8x}=\frac{2}{\sin 8x}$$

$f(x)$ na intervalu $\left\langle 0,\frac{\pi}{4}\right\rangle$ poprima najmanju vrijednost kad $\sin 8x$ poprima najveću.

$x\in\left\langle 0,\frac{\pi}{4}\right\rangle \implies 8x\in\langle 0,2\pi\rangle$

$\sin 8x$ poprima najveću vrijednost za $8x=\frac{\pi}{2}$, tj. $x=\frac{\pi}{16}$ pa je najveća vrijednost funkcije $f$ :

$$f\left(\frac{\pi}{16}\right)=\frac{2}{\sin\frac{\pi}{2}}=\frac{2}{1}=2.$$