Zadatak 28. Izračunaj bez uporabe računala:

1) $\sin\frac{\pi}{12}\cdot\sin\frac{5\pi}{12}$;

2) $\sin^4\frac{5\pi}{12}-\sin^4\frac{11\pi}{12}$;

3) $1-8\sin^2\frac{\pi}{16}\cdot\cos^2\frac{\pi}{16}$;

4) $\left(\cos^2\frac{13\pi}{18}-\cos^2\frac{7\pi}{9}\right)^2+\cos^2\frac{17\pi}{18}$.

Rješenje.

1) $\sin\frac{\pi}{12}\cdot\sin\frac{5\pi}{12}=\sin\frac{\pi}{12}\cdot\cos\left(\frac{\pi}{2}-\frac{5\pi}{12}\right)=\sin\frac{\pi}{12}\cdot\cos\frac{\pi}{12}=\frac{1}{2}\cdot\sin 2\cdot\frac{\pi}{12}$
$=\frac{1}{2}\cdot\sin\frac{\pi}{6}=\frac{1}{4}$;

2) $\sin^4\frac{5\pi}{12}-\sin^4\frac{11\pi}{12}=\cos^4\left(\frac{\pi}{2}-\frac{5\pi}{12}\right)-\sin^4\left(\pi-\frac{11\pi}{12}\right)$
$=\cos^4\frac{\pi}{12}-\sin^4\frac{\pi}{12}=\left(\cos^2\frac{\pi}{12}-\sin^2\frac{\pi}{12}\right)\cdot\left(\cos^2\frac{\pi}{12}+\sin^2\frac{\pi}{12}\right)$
$=\cos^2\frac{\pi}{12}-\sin^2\frac{\pi}{12}=\cos 2\cdot\frac{\pi}{12}=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$;

3) $1-8\sin^2\frac{\pi}{16}\cdot\cos^2\frac{\pi}{16}=1-2\sin^2\frac{\pi}{8}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$;

4) $\left(\cos^2\frac{13\pi}{18}-\cos^2\frac{7\pi}{9}\right)^2+\cos^2\frac{17\pi}{18}$
$=\left[\cos^2\left(\pi-\frac{13\pi}{18}\right)-\cos^2\left(\pi-\frac{7\pi}{9}\right)\right]^2+\cos^2\left(\pi-\frac{17\pi}{18}\right)$
$=\left(\cos^2\frac{5\pi}{18}-\cos^2\frac{2\pi}{9}\right)^2+\cos^2\frac{\pi}{18}$
$=\left[\sin^2\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-\cos^2\frac{2\pi}{9}\right]^2+\sin^2\left(\frac{\pi}{2}-\frac{\pi}{18}\right)$
$=\left(\sin^2\frac{2\pi}{9}-\cos^2\frac{2\pi}{9}\right)^2+\sin^2\frac{4\pi}{9}=\cos^2\frac{4\pi}{9}+\sin^2\frac{4\pi}{9}=1$.