Zadatak 31. Bez uporabe računala izračunaj:

1) $\sin 18^\circ$; **2)** $\cos 9^\circ$.

Rješenje. **1)**

$$\begin{aligned}
\sin 36^\circ &= \cos(90^\circ - 36^\circ)\\
\sin(2\cdot 18)^\circ &= \cos 54^\circ\\
\sin(2\cdot 18)^\circ &= \cos(3\cdot 18)^\circ\\
2\sin 18^\circ\cos 18^\circ &= 4\cos^3 18^\circ - 3\cos 18^\circ \quad /:\cos 18^\circ\\
2\sin 18^\circ &= 4\cos^2 18^\circ - 3\\
2\sin 18^\circ - 4(1-\sin^2 18^\circ) + 3 &= 0\\
4\sin^2 18^\circ + 2\sin 18^\circ - 1 &= 0\\
(\sin 18^\circ)_{1,2} = \frac{-2\pm\sqrt{4+16}}{8} &= \frac{-2\pm 2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4}\\
\Longrightarrow\ (\text{jer } \sin 18^\circ > 0) &\Longrightarrow \sin 18^\circ = \frac{\sqrt{5}-1}{4}
\end{aligned}$$

2) Iz $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ imamo:

$$\begin{aligned}
\cos 18^\circ &= \sqrt{1-\sin^2 18^\circ} = \sqrt{1-\frac{5-2\sqrt{5}+1}{16}} = \sqrt{1-\frac{6-2\sqrt{5}}{16}}\\
&= \sqrt{\frac{10+2\sqrt{5}}{16}} = \frac{\sqrt{10+2\sqrt{5}}}{4}
\end{aligned}$$

$$\begin{aligned}
\cos 9^\circ &= \sqrt{\frac{1+\cos 18^\circ}{2}} = \sqrt{\frac{1+\frac{\sqrt{10+2\sqrt{5}}}{4}}{2}} = \sqrt{\frac{4+\sqrt{10+2\sqrt{5}}}{8}}\\
&= \sqrt{\frac{8+2\sqrt{10+2\sqrt{5}}}{16}} = \frac{1}{4}\sqrt{8+2\sqrt{10+2\sqrt{5}}}
\end{aligned}$$