

Zadatak 3. Napiši u obliku umnoška:

- 1) $\sqrt{3} - 2 \sin x$; 2) $\sqrt{2} + 2 \sin x$;
 3) $1 - \sqrt{2} \cos x$; 4) $1 - 4 \cos^2 x$;
 5) $4 \sin^2 x - 3$.

Rješenje.

1)

$$\begin{aligned}\sqrt{3} - 2 \sin x &= 2 \left(\frac{\sqrt{3}}{2} - \sin x \right) = 2 \left(\sin \frac{\pi}{3} - \sin x \right) = 2 \cdot 2 \cos \frac{\frac{\pi}{3} + x}{2} \sin \frac{\frac{\pi}{3} - x}{2} \\ &= 4 \cos \left(\frac{\pi}{6} + \frac{x}{2} \right) \sin \left(\frac{\pi}{6} - \frac{x}{2} \right)\end{aligned}$$

2)

$$\begin{aligned}\sqrt{2} + 2 \sin x &= 2 \left(\frac{\sqrt{2}}{2} + \sin x \right) = 2 \left(\sin \frac{\pi}{4} + \sin x \right) = 2 \cdot 2 \sin \frac{\frac{\pi}{4} + x}{2} \cos \frac{\frac{\pi}{4} - x}{2} \\ &= 4 \sin \left(\frac{\pi}{8} + \frac{x}{2} \right) \cos \left(\frac{\pi}{8} - \frac{x}{2} \right)\end{aligned}$$

3)

$$\begin{aligned}1 - \sqrt{2} \cos x &= \sqrt{2} \left(\frac{\sqrt{2}}{2} - \cos x \right) = \sqrt{2} \left(\cos \frac{\pi}{4} - \cos x \right) = -2\sqrt{2} \sin \frac{\frac{\pi}{4} + x}{2} \sin \frac{\frac{\pi}{4} - x}{2} \\ &= -2\sqrt{2} \sin \left(\frac{\pi}{8} + \frac{x}{2} \right) \sin \left(\frac{\pi}{8} - \frac{x}{2} \right)\end{aligned}$$

4)

$$\begin{aligned}1 - 4 \cos^2 x &= (1 - 2 \cos x)(1 + 2 \cos x) = 2 \cdot \left(\frac{1}{2} - \cos x \right) \cdot 2 \cdot \left(\frac{1}{2} + \cos x \right) \\ &= 4 \cdot \left(\cos \frac{\pi}{3} - \cos x \right) \left(\cos \frac{\pi}{3} + \cos x \right) \\ &= 4 \cdot (-2) \sin \frac{\frac{\pi}{3} + x}{2} \sin \frac{\frac{\pi}{3} - x}{2} \cdot 2 \cos \frac{\frac{\pi}{3} + x}{2} \cos \frac{\frac{\pi}{3} - x}{2} \\ &= -4 \sin \left(\frac{\pi}{3} + x \right) \cdot \sin \left(\frac{\pi}{3} - x \right)\end{aligned}$$

5)

$$\begin{aligned}4 \sin^2 x - 3 &= 4 \left(\sin^2 x - \frac{3}{4} \right) = 4 \left(\sin x - \frac{\sqrt{3}}{2} \right) \left(\sin x + \frac{\sqrt{3}}{2} \right) = 4 \left(\sin x - \sin \frac{\pi}{3} \right) \left(\sin x + \sin \frac{\pi}{3} \right) \\ &= 4 \cdot 2 \cos \left(\frac{x}{2} + \frac{\pi}{6} \right) \sin \left(\frac{x}{2} - \frac{\pi}{6} \right) \cdot 2 \sin \left(\frac{x}{2} + \frac{\pi}{6} \right) \cos \left(\frac{x}{2} - \frac{\pi}{6} \right) \\ &= 4 \cdot \sin 2 \left(\frac{x}{2} + \frac{\pi}{6} \right) \sin 2 \left(\frac{x}{2} - \frac{\pi}{6} \right) = 4 \cdot \sin \left(x + \frac{\pi}{3} \right) \sin \left(x - \frac{\pi}{3} \right)\end{aligned}$$