Zadatak 7. Izračunaj bez uporabe računala:

1) $\cos 105^\circ \cdot \cos 75^\circ$;

2) $\cos 37^\circ 30' \cdot \cos 7^\circ 30'$;

3) $\sin \frac{\pi}{24} \cdot \sin \frac{5\pi}{24}$;

4) $\cos 52^\circ 30' \cdot \sin 7^\circ 30'$.

Rješenje. **1)**

$$\cos 105^\circ \cdot \cos 75^\circ = \frac{1}{2}[\cos(105^\circ + 75^\circ) + \cos(105^\circ - 75^\circ)] = \frac{1}{2}(\cos 180^\circ + \cos 30^\circ)$$
$$= \frac{1}{2}\left(-1 + \frac{\sqrt{3}}{2}\right) = \frac{1}{4}(-2 + \sqrt{3})$$

2)

$$\cos 37^\circ 30' \cdot \cos 7^\circ 30' = \frac{1}{2}[\cos(37^\circ 30' + 7^\circ 30') + \cos(37^\circ 30' - 7^\circ 30')]$$
$$= \frac{1}{2}(\cos 45^\circ + \cos 30^\circ) = \frac{1}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right) = \frac{1}{4}(\sqrt{2} + \sqrt{3})$$

3)

$$\sin \frac{\pi}{24} \cdot \sin \frac{5\pi}{24} = \frac{1}{2}\left[\cos\left(\frac{\pi}{24} - \frac{5\pi}{24}\right) - \cos\left(\frac{\pi}{24} + \frac{5\pi}{24}\right)\right] = \frac{1}{2}\left(\cos \frac{\pi}{6} - \cos \frac{\pi}{4}\right)$$
$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\right) = \frac{1}{4}(\sqrt{3} - \sqrt{2})$$

4)

$$\cos 52^\circ 30' \cdot \sin 7^\circ 30' = \frac{1}{2}[\sin(52^\circ 30' + 7^\circ 30') - \sin(52^\circ 30' - 7^\circ 30')] = \frac{1}{2}(\sin 60^\circ - \sin 45^\circ)$$
$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\right) = \frac{1}{4}(\sqrt{3} - \sqrt{2})$$

$$\frac{\sqrt{3} - \sqrt{2}}{4}.$$