Zadatak 11. Dokaži:

1) $\dfrac{\sin x+\sin 3x+\sin 5x+\sin 7x}{\cos x+\cos 3x+\cos 5x+\cos 7x}=\operatorname{tg} 4x$;

2) $\dfrac{\sin x+\sin 3x+\ldots+\sin(2n-1)x}{\cos x+\cos 3x+\ldots+\cos(2n-1)x}=\operatorname{tg} nx$.

Rješenje. **1)**

$$\frac{\sin x+\sin 3x+\sin 5x+\sin 7x}{\cos x+\cos 3x+\cos 5x+\cos 7x}=\operatorname{tg} 4x$$

$$\frac{(\sin x+\sin 7x)+(\sin 3x+\sin 5x)}{(\cos x+\cos 7x)+(\cos 3x+\cos 5x)}=\operatorname{tg} 4x$$

$$\frac{2\sin\frac{x+7x}{2}\cos\frac{x-7x}{2}+2\sin\frac{3x+5x}{2}\cos\frac{3x-5x}{2}}{2\cos\frac{x+7x}{2}\cos\frac{x-7x}{2}+2\cos\frac{3x+5x}{2}\cos\frac{3x-5x}{2}}=\operatorname{tg} 4x$$

$$\frac{2\sin 4x\cos(-3x)+2\sin 4x\cos(-x)}{2\cos 4x\cos(-3x)+2\cos 4x\cos(-x)}=\operatorname{tg} 4x$$

$$\frac{2\sin 4x(\cos 3x+\cos x)}{2\cos 4x(\cos 3x+\cos x)}=\operatorname{tg} 4x$$

$$\operatorname{tg} 4x=\operatorname{tg} 4x$$

2) Dokazuje se analogno navedenom rješenju ili pak matematičkom indukcijom.

$$\frac{\sin x+\sin 3x+\ldots+\sin(2n-1)x}{\cos x+\cos 3x+\ldots+\cos(2n-1)x}=\operatorname{tg} nx$$

$$\frac{(\sin x+\sin(2n-1)x)+(\sin 3x+\sin(2n-3)x)+\ldots}{\cos x+\cos(2n-1)x)+(\cos 3x+\cos(2n-3)x)+\ldots}=\operatorname{tg} nx$$

$$\frac{2\sin\frac{x+(2n-1)x}{2}\cos\frac{x-(2n-1)x}{2}+2\sin\frac{3x+(2n-3)x}{2}\cos\frac{3x-(2n-3)x}{2}+\ldots+2\sin\frac{(n-1)x+(n+1)x}{2}\cos\frac{(n-1)x-(n+1)x}{2}}{2\cos\frac{x+(2n-1)x}{2}\cos\frac{x-(2n-1)x}{2}+2\cos\frac{3x+(2n-3)x}{2}\cos\frac{3x-(2n-3)x}{2}+\ldots+2\cos\frac{(n-1)x+(n+1)x}{2}\cos\frac{(n-1)x-(n+1)x}{2}}=\operatorname{tg} nx$$

$$\frac{2\sin nx\cos(-n+1)x+2\sin nx\cos(-n+3)x+\ldots+2\sin nx\cos(-x)}{2\cos nx\cos(-n+1)x+2\cos nx\cos(-n+3)x+\ldots+2\cos nx\cos(-x)}=\operatorname{tg} nx$$

$$\frac{2\sin nx[\cos(-n+1)x+\cos(-n+3)x+\ldots+\cos(-x)]}{2\cos nx[\cos(-n+1)x+\cos(-n+3)x+\ldots+\cos(-x)]}=\operatorname{tg} nx$$

$$\operatorname{tg} nx=\operatorname{tg} nx$$