

Zadatak 5.

Koliko je

$$\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11} ?$$

Rješenje.

Svaki ćemo pribrojnik pomnožiti i podijeliti s $2 \sin \frac{\pi}{11}$, te primijeniti formule za pretvorbu umnoška u zbroj trigonometrijskih funkcija:

$$\begin{aligned} & \cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11} \\ &= \frac{\cos \frac{\pi}{11} \cdot \sin \frac{\pi}{11}}{\sin \frac{\pi}{11}} + \frac{\cos \frac{3\pi}{11} \cdot \sin \frac{\pi}{11}}{\sin \frac{\pi}{11}} + \frac{\cos \frac{5\pi}{11} \cdot \sin \frac{\pi}{11}}{\sin \frac{\pi}{11}} + \frac{\cos \frac{7\pi}{11} \cdot \sin \frac{\pi}{11}}{\sin \frac{\pi}{11}} + \frac{\cos \frac{9\pi}{11} \cdot \sin \frac{\pi}{11}}{\sin \frac{\pi}{11}} \\ &= \frac{\frac{1}{2} \left[\sin \left(\frac{\pi}{11} + \frac{\pi}{11} \right) - \sin \left(\frac{\pi}{11} - \frac{\pi}{11} \right) \right]}{\sin \frac{\pi}{11}} + \frac{\frac{1}{2} \left[\sin \left(\frac{3\pi}{11} + \frac{\pi}{11} \right) - \sin \left(\frac{3\pi}{11} - \frac{\pi}{11} \right) \right]}{\sin \frac{\pi}{11}} \\ &\quad + \frac{\frac{1}{2} \left[\sin \left(\frac{5\pi}{11} + \frac{\pi}{11} \right) - \sin \left(\frac{5\pi}{11} - \frac{\pi}{11} \right) \right]}{\sin \frac{\pi}{11}} + \frac{\frac{1}{2} \left[\sin \left(\frac{7\pi}{11} + \frac{\pi}{11} \right) - \sin \left(\frac{7\pi}{11} - \frac{\pi}{11} \right) \right]}{\sin \frac{\pi}{11}} \\ &\quad + \frac{\frac{1}{2} \left[\sin \left(\frac{9\pi}{11} + \frac{\pi}{11} \right) - \sin \left(\frac{9\pi}{11} - \frac{\pi}{11} \right) \right]}{\sin \frac{\pi}{11}} \\ &= \frac{\sin \frac{2\pi}{11} + \sin \frac{4\pi}{11} - \sin \frac{2\pi}{11} + \sin \frac{6\pi}{11} - \sin \frac{4\pi}{11} + \sin \frac{8\pi}{11} - \sin \frac{6\pi}{11} + \sin \frac{10\pi}{11} - \sin \frac{8\pi}{11}}{2 \sin \frac{\pi}{11}} \\ &= \frac{\sin \frac{10\pi}{11}}{2 \sin \frac{\pi}{11}} = \frac{\sin \left(\pi - \frac{10\pi}{11} \right)}{2 \sin \frac{\pi}{11}} = \frac{\sin \frac{\pi}{11}}{2 \sin \frac{\pi}{11}} = \frac{1}{2} \end{aligned}$$