Zadatak 6. Izračunaj bez uporabe računala:

$$\sin^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}.$$

Rješenje.

$$\begin{aligned}
&\sin^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}\\
&=\sin^4\frac{\pi}{8}+\left[-\cos\left(\pi-\frac{3\pi}{8}\right)\right]^4+\sin^4\frac{5\pi}{8}+\left[-\cos\left(\pi-\frac{7\pi}{8}\right)\right]^4\\
&=\sin^4\frac{\pi}{8}+\left(-\cos\frac{5\pi}{8}\right)^4+\sin^4\frac{5\pi}{8}+\left(-\cos\frac{\pi}{8}\right)^4=\sin^4\frac{\pi}{8}+\cos^4\frac{5\pi}{8}+\sin^4\frac{5\pi}{8}+\cos^4\frac{\pi}{8}\\
&=\left(\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right)^2-2\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}+\left(\sin^2\frac{5\pi}{8}+\cos^2\frac{5\pi}{8}\right)^2-2\sin^2\frac{5\pi}{8}\cdot\cos^2\frac{5\pi}{8}\\
&=1-\frac{1}{2}\sin^2\frac{\pi}{4}+1-\frac{1}{2}\sin^2\frac{5\pi}{4}=2-\frac{1}{2}\cdot\left(\frac{\sqrt{2}}{2}\right)^2-\frac{1}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)^2=2-\frac{1}{2}\cdot\frac{2}{4}-\frac{1}{2}\cdot\frac{2}{4}=\frac{3}{2}.
\end{aligned}$$