

Zadatak 8. Dokaži da je $\cos 18^\circ = \sqrt{\frac{5+\sqrt{5}}{8}}$.

$$\textbf{Rješenje.} \quad \cos 18^\circ = \cos \frac{\pi}{10} = \sqrt{\frac{5+\sqrt{5}}{8}}$$

$$\begin{aligned}\sin 18^\circ \cdot \cos 36^\circ &= \frac{2 \sin 18^\circ \cdot \cos 18^\circ \cdot \cos 36^\circ}{2 \cos 18^\circ} = \frac{\sin 36^\circ \cdot \cos 36^\circ}{2 \cos 18^\circ} \\ &= \frac{\sin(2 \cdot 36^\circ)}{4 \cos 18^\circ} = \frac{\cos(90^\circ - 72^\circ)}{4 \cos 18^\circ} = \frac{1}{4};\end{aligned}$$

No,

$$\cos 36^\circ = \cos(2 \cdot 18^\circ) = \cos^2 18^\circ - \sin^2 18^\circ = 1 - 2 \sin^2 18^\circ,$$

te tako imamo jednadžbu

$$\sin 18^\circ \cdot \cos 36^\circ - \frac{1}{4} = 0$$

$$\sin 18^\circ \cdot (1 - 2 \sin^2 18^\circ) - \frac{1}{4} = 0$$

$$\sin 18^\circ - 2 \sin^3 18^\circ - \frac{1}{4} = 0 \quad / \cdot 4$$

$$8 \sin^3 18^\circ - 4 \sin 18^\circ + 1 = 0.$$

Uvođenjem supstitucije $\sin 18^\circ = x$ dobije se:

$$8x^3 - 4x + 1 = 0$$

$$8x^3 - 2x - 2x + 1 = 0$$

$$2x(4x^2 - 1) - (2x - 1) = 0$$

$$2x(2x - 1)(2x + 1) - (2x - 1) = 0$$

$$(2x - 1)[2x(2x + 1) - 1] = 0$$

$$(2x - 1)(4x^2 + 2x - 1) = 0$$

Rješenja su:

$$x_1 = \frac{1}{2}$$

$$x_{2,3} = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

Rješenje $\sin 18^\circ = \frac{1}{2}$ ne dolazi u obzir, jer je $\sin 30^\circ = \frac{1}{2}$, a od preostala dva rješenja uzimamo ono pozitivno. Sad se lako može naći

$$\begin{aligned}\cos 18^\circ &= \sqrt{1 - \sin^2 18^\circ} = \sqrt{1 - \left(\frac{-1 + \sqrt{5}}{4}\right)^2} = \sqrt{1 - \frac{1 - 2\sqrt{5} + 5}{16}} \\ &= \sqrt{\frac{16 - 6 + 2\sqrt{5}}{16}} = \sqrt{\frac{5 + \sqrt{5}}{8}}.\end{aligned}$$