Zadatak 10. Izračunaj: $\text{tg}^2\frac{\pi}{12}+\text{tg}^2\frac{3\pi}{12}+\text{tg}^2\frac{5\pi}{12}$.

Rješenje.

$$\text{tg}^2\frac{\pi}{12}+\text{tg}^2\frac{3\pi}{12}+\text{tg}^2\frac{5\pi}{12}=\text{tg}^2\frac{\pi}{12}+\text{tg}^2\frac{\pi}{4}+\text{ctg}^2\left(\frac{\pi}{2}-\frac{5\pi}{12}\right)=\text{tg}^2\frac{\pi}{12}+1+\text{ctg}^2\frac{\pi}{12}$$

$$=\frac{\sin^2\frac{\pi}{12}}{\cos^2\frac{\pi}{12}}+\frac{\cos^2\frac{\pi}{12}}{\sin^2\frac{\pi}{12}}+1=\frac{\frac{1-\cos\frac{\pi}{6}}{2}}{\frac{1+\cos\frac{\pi}{6}}{2}}+\frac{\frac{1+\cos\frac{\pi}{6}}{2}}{\frac{1-\cos\frac{\pi}{6}}{2}}+1=\frac{\frac{1-\frac{\sqrt{3}}{2}}{2}}{\frac{1+\frac{\sqrt{3}}{2}}{2}}+\frac{\frac{1+\frac{\sqrt{3}}{2}}{2}}{\frac{1-\frac{\sqrt{3}}{2}}{2}}+1$$

$$=\frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}+\frac{\frac{2+\sqrt{3}}{2}}{\frac{2-\sqrt{3}}{2}}+1=\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{2+\sqrt{3}}{2-\sqrt{3}}+1=\frac{(2-\sqrt{3})^2+(2+\sqrt{3})^2}{4-3}+1$$

$$=\frac{4-4\sqrt{3}+3+4+4\sqrt{3}+3}{4-3}+1=14+1=15$$