Zadatak 11. Koliko je $\operatorname{tg}^2 36^\circ \cdot \operatorname{tg}^2 72^\circ$?

Rješenje.

$$\operatorname{tg}^2 36^\circ \cdot \operatorname{tg}^2 72^\circ = \frac{\sin^2 36^\circ \cdot \sin^2 72^\circ}{\cos^2 36^\circ \cdot \cos^2 72^\circ} = \frac{\sin^2 36^\circ \cdot \sin^2 72^\circ - \cos^2 36^\circ \cdot \cos^2 72^\circ}{\cos^2 36^\circ \cdot \cos^2 72^\circ} + 1$$

$$= \frac{(\sin 36^\circ \cdot \sin 72^\circ - \cos 36^\circ \cdot \cos 72^\circ)(\sin 36^\circ \cdot \sin 72^\circ + \cos 36^\circ \cdot \cos 72^\circ)}{\cos^2 36^\circ \cdot \cos^2 72^\circ} + 1$$

$$= \frac{-\cos(72^\circ + 36^\circ) \cdot \cos(72^\circ - 36^\circ)}{\cos^2 36^\circ \cdot \cos^2 72^\circ} + 1 = \frac{-\cos 108^\circ \cdot \cos 36^\circ}{\cos^2 36^\circ \cdot \cos^2 72^\circ} + 1$$

$$= \frac{-[-\cos(180^\circ - 108^\circ)]}{\cos 36^\circ \cdot \cos^2 72^\circ} + 1 = \frac{\cos 72^\circ}{\cos 36^\circ \cdot \cos^2 72^\circ} + 1 = \frac{1}{\cos 36^\circ \cdot \cos 72^\circ} + 1$$

$$= \frac{2 \sin 36^\circ}{2 \sin 36^\circ \cos 36^\circ \cdot \cos 72^\circ} + 1 = \frac{2 \sin 36^\circ}{\sin 72^\circ \cdot \cos 72^\circ} + 1 = \frac{2 \sin 36^\circ}{\frac{1}{2} \sin 144^\circ} + 1$$

$$= \frac{4 \sin 36^\circ}{\sin(180^\circ - 144^\circ)} + 1 = \frac{4 \sin 36^\circ}{\sin 36^\circ} + 1 = 4 + 1 = 5$$