

**Zadatak 13.** Dokaži:  $\operatorname{tg} \alpha \cdot \operatorname{tg}\left(\frac{\pi}{3} - \alpha\right) \cdot \operatorname{tg}\left(\frac{\pi}{3} + \alpha\right) = \operatorname{tg} 3\alpha$ .

*Rješenje.*

$$\begin{aligned} \operatorname{tg} \alpha \cdot \operatorname{tg}\left(\frac{\pi}{3} - \alpha\right) \cdot \operatorname{tg}\left(\frac{\pi}{3} + \alpha\right) &= \operatorname{tg} 3\alpha \\ \operatorname{tg} \alpha \cdot \frac{\operatorname{tg} \frac{\pi}{3} - \operatorname{tg} \alpha}{1 + \operatorname{tg} \frac{\pi}{3} \operatorname{tg} \alpha} \cdot \frac{\operatorname{tg} \frac{\pi}{3} + \operatorname{tg} \alpha}{1 - \operatorname{tg} \frac{\pi}{3} \operatorname{tg} \alpha} &= \operatorname{tg} 3\alpha \\ \operatorname{tg} \alpha \cdot \frac{\sqrt{3} - \operatorname{tg} \alpha}{1 + \sqrt{3} \operatorname{tg} \alpha} \cdot \frac{\sqrt{3} + \operatorname{tg} \alpha}{1 - \sqrt{3} \operatorname{tg} \alpha} &= \operatorname{tg} 3\alpha \\ \frac{\operatorname{tg} \alpha \cdot (3 - \operatorname{tg}^2 \alpha)}{1 - 3 \operatorname{tg}^2 \alpha} &= \frac{\operatorname{tg} 2\alpha + \operatorname{tg} \alpha}{1 - \operatorname{tg} 2\alpha \operatorname{tg} \alpha} \\ \frac{\operatorname{tg} \alpha \cdot (3 - \operatorname{tg}^2 \alpha)}{1 - 3 \operatorname{tg}^2 \alpha} &= \frac{\frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} + \operatorname{tg} \alpha}{1 - \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} \operatorname{tg} \alpha} \\ \frac{\operatorname{tg} \alpha \cdot (3 - \operatorname{tg}^2 \alpha)}{1 - 3 \operatorname{tg}^2 \alpha} &= \frac{\frac{2 \operatorname{tg} \alpha + \operatorname{tg} \alpha - \operatorname{tg}^3 \alpha}{1 - \operatorname{tg}^2 \alpha}}{\frac{1 - \operatorname{tg}^2 \alpha - 2 \operatorname{tg}^2 \alpha}{1 - \operatorname{tg}^2 \alpha}} \\ \frac{\operatorname{tg} \alpha \cdot (3 - \operatorname{tg}^2 \alpha)}{1 - 3 \operatorname{tg}^2 \alpha} &= \frac{3 \operatorname{tg} \alpha - \operatorname{tg}^3 \alpha}{1 - 3 \operatorname{tg}^2 \alpha} \\ \frac{\operatorname{tg} \alpha \cdot (3 - \operatorname{tg}^2 \alpha)}{1 - 3 \operatorname{tg}^2 \alpha} &= \frac{\operatorname{tg} \alpha \cdot (3 - \operatorname{tg}^2 \alpha)}{1 - 3 \operatorname{tg}^2 \alpha} \end{aligned}$$