Zadatak 16. Ako za kutove α, β i γ trokuta ABC vrijedi $\sin\alpha = 4\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$, taj je trokut jednakokračan. Dokaži!

Rješenje.

$$\sin\alpha = 4\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\cos\frac{\gamma}{2} \quad \Big/ : 2\sin\frac{\alpha}{2}$$

$$\frac{\sin\alpha}{2\sin\frac{\alpha}{2}} = 2\sin\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$$

$$\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}} = 2\sin\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$$

$$\cos\frac{\alpha}{2} = 2\sin\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$$

$$\cos\frac{\alpha}{2} = \sin\frac{\beta+\gamma}{2} + \sin\frac{\beta-\gamma}{2} \qquad \left(\frac{\beta+\gamma}{2} = \frac{\pi}{2} - \frac{\alpha}{2}\right)$$

$$\cos\frac{\alpha}{2} = \sin\left(\frac{\pi}{2} - \frac{\alpha}{2}\right) + \sin\frac{\beta-\gamma}{2}$$

$$\cos\frac{\alpha}{2} = \cos\frac{\alpha}{2} + \sin\frac{\beta-\gamma}{2}$$

$$0 = \sin\frac{\beta-\gamma}{2} \qquad \Longrightarrow \frac{\beta-\gamma}{2} = 0 \Longrightarrow \beta = \gamma.$$