Zadatak 17. Ako je $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\alpha+\beta+\gamma=\pi$ te $\cos\alpha+\cos\beta+\cos\gamma=\frac{3}{2}$, odredi α, β i γ.

Rješenje.

$$\alpha+\beta=\pi-\gamma \implies \cos(\alpha+\beta)=\cos\pi\cos\gamma+\sin\pi\sin\gamma=-\cos\gamma$$
$$\implies \cos\gamma=-\cos(\alpha+\beta)$$

$$\cos\alpha+\cos\beta-\cos(\alpha+\beta)=\frac{3}{2}$$
$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-2\cos^2\frac{\alpha+\beta}{2}+1=\frac{3}{2}$$
$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-2\cos^2\frac{\alpha+\beta}{2}-\frac{1}{2}=0 \quad \Big/\cdot(-2)$$
$$4\cos^2\frac{\alpha+\beta}{2}-4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+1=0$$
$$\cos\frac{\alpha+\beta}{2}=\frac{4\cos\frac{\alpha-\beta}{2}\pm\sqrt{16\cos^2\frac{\alpha-\beta}{2}-16}}{8}$$
$$\implies 16\cos^2\frac{\alpha-\beta}{2}-16\geqslant 0 \quad \Big/:16$$
$$\cos^2\frac{\alpha-\beta}{2}-1\geqslant 0$$
$$\cos^2\frac{\alpha-\beta}{2}\geqslant 1 \implies \cos^2\frac{\alpha-\beta}{2}=1 \text{ (jedino moguće)}$$
$$\implies \cos\frac{\alpha-\beta}{2}=\pm 1$$

$$\cos\frac{\alpha-\beta}{2}=-1 \implies \frac{\alpha-\beta}{2}=\pi \text{ nije rješenje}$$
$$\cos\frac{\alpha-\beta}{2}=1 \implies \alpha-\beta=0 \implies \alpha=\beta$$
$$\implies \cos\frac{2\alpha}{2}=\frac{4\cos 0\pm 0}{8}=\frac{4}{8}=\frac{1}{2}$$
$$\cos\alpha=\frac{1}{2} \implies \alpha=\frac{\pi}{3}, \quad \beta=\alpha=\frac{\pi}{3}, \quad \gamma=\pi-\alpha-\beta=\frac{\pi}{3}$$