

Zadatak 20. Ako je $\alpha + \beta + \gamma = \pi$, dokaži da je onda

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2(1 + \cos \alpha \cdot \cos \beta \cdot \cos \gamma).$$

Rješenje.

$$\begin{aligned}\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma &= \frac{1 - \cos 2\alpha}{2} \\&+ \frac{1 - \cos 2\beta}{2} + 1 - \cos^2 \gamma \\&= 2 - \frac{\cos 2\alpha + \cos 2\beta}{2} - \cos^2 \gamma \\&= 2 - \cos(\alpha + \beta) \cdot \cos(\alpha - \beta) - \cos^2(\pi - (\alpha + \beta)) \\&= 2 - \cos(\alpha + \beta) \cdot \cos(\alpha - \beta) - \cos^2(\alpha + \beta) \\&= 2 - \cos(\alpha + \beta)[\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\&= 2 - \cos(\pi - \gamma) \cdot 2 \cos \alpha \cdot \cos \beta \\&= 2 + 2 \cos \alpha \cdot \cos \beta \cdot \cos \gamma.\end{aligned}$$