Zadatak 21. Ako je $\cos\alpha+\cos\beta=a$, $\sin\alpha+\sin\beta=b$, dokaži da je tada

$$\cos(\alpha+\beta)=\frac{a^2-b^2}{a^2+b^2};\qquad \sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}.$$

Rješenje.

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=a,$$
$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=b,$$

Dijeljenjem ovih jednakosti dobivamo $\operatorname{tg}\frac{\alpha+\beta}{2}=\frac{b}{a}$. Sad je

$$\cos(\alpha+\beta)=\frac{1-\operatorname{tg}^2\frac{\alpha-b}{2}}{1+\operatorname{tg}^2\frac{\alpha-b}{2}}=\frac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}}=\frac{a^2-b^2}{a^2+b^2},$$
$$\sin(\alpha+\beta)=\frac{2\operatorname{tg}\frac{\alpha-b}{2}}{1+\operatorname{tg}^2\frac{\alpha-b}{2}}=\frac{2\cdot\frac{b}{a}}{1+\frac{b^2}{a^2}}=\frac{2ab}{a^2+b^2},$$