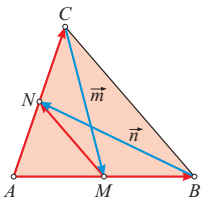
Zadatak 9. U trokutu ABC točke M i N polovišta su stranica $\overline{AB}$ i $\overline{AC}$. Prikaži vektore $\overrightarrow{AB}$, $\overrightarrow{AC}$ i $\overrightarrow{MN}$ kao linearnu kombinaciju vektora $\vec{m} = \overrightarrow{CM}$ i $\vec{n} = \overrightarrow{BN}$.

Rješenje.

$$\overrightarrow{AB} = 2\overrightarrow{AM}$$
$$\overrightarrow{AB} = 2(\overrightarrow{AC} + \overrightarrow{CM})$$
$$\overrightarrow{AB} = 2\overrightarrow{AC} + 2\vec{m}$$
$$\overrightarrow{AB} = 2 \cdot 2\overrightarrow{AN} + 2\vec{m}$$
$$\overrightarrow{AB} = 4(\overrightarrow{AB} + \vec{n}) + 2\vec{m}$$
$$-3\overrightarrow{AB} = 4\vec{n} + 2\vec{m}$$
$$\overrightarrow{AB} = -\frac{4}{3}\vec{n} - \frac{2}{3}\vec{m}$$



$$\overrightarrow{AC} = 2\overrightarrow{AN}$$
$$\overrightarrow{AC} = 2(\overrightarrow{AB} + \overrightarrow{BN})$$
$$\overrightarrow{AC} = 2\overrightarrow{AB} + 2\vec{n}$$
$$\overrightarrow{AC} = 2 \cdot 2\overrightarrow{AM} + 2\vec{n}$$
$$\overrightarrow{AC} = 4(\overrightarrow{AC} + \vec{m}) + 2\vec{n}$$
$$-3\overrightarrow{AC} = 4\vec{m} + 2\vec{n}$$
$$\overrightarrow{AC} = -\frac{4}{3}\vec{m} - \frac{2}{3}\vec{n}$$

$$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$$
$$\overrightarrow{MN} = \frac{1}{2}\overrightarrow{BA} + \frac{1}{2}\overrightarrow{AC}$$
$$\overrightarrow{MN} = -\frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC}$$
$$\overrightarrow{MN} = -\frac{1}{2}\left(-\frac{4}{3}\vec{n} - \frac{2}{3}\vec{m}\right) + \frac{1}{2}\left(-\frac{4}{3}\vec{m} - \frac{2}{3}\vec{n}\right)$$
$$\overrightarrow{MN} = \frac{2}{3}\vec{n} + \frac{1}{3}\vec{m} - \frac{2}{3}\vec{m} - \frac{1}{3}\vec{n}$$
$$\overrightarrow{MN} = \frac{1}{3}\vec{n} - \frac{1}{3}\vec{m}$$