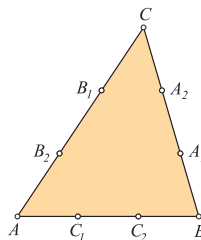


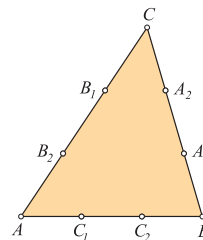
Zadatak 12. Točkama $A_1, A_2, B_1, B_2, C_1, C_2$ stranice trokuta podijeljene su na tri sukladna dijela. Dokaži:

- 1) $\overrightarrow{A_1C_2} + \overrightarrow{C_1B_2} + \overrightarrow{B_1A_2} = \vec{0}$;
- 2) $\overrightarrow{C_1A_2} = \overrightarrow{C_2B_1} + \overrightarrow{B_2A_1}$;
- 3) $\overrightarrow{A_1B_1} = \overrightarrow{C_2B_2}$.



Rješenje.

$$\begin{aligned} 1) \quad \overrightarrow{A_1C_2} + \overrightarrow{C_1B_2} + \overrightarrow{B_1A_2} &= \overrightarrow{A_1B} + \overrightarrow{BC_2} + \overrightarrow{C_1A} + \overrightarrow{AB_2} + \overrightarrow{B_1C} + \overrightarrow{CA_2} = \\ &= (\overrightarrow{C_1A} + \overrightarrow{BC_2}) + (\overrightarrow{AB_2} + \overrightarrow{B_1C}) + (\overrightarrow{A_1B} + \overrightarrow{CA_2}) = \frac{2}{3}(\overrightarrow{BA} + \overrightarrow{AC} + \overrightarrow{CB}) = \vec{0}. \end{aligned}$$



$$2) \quad \overrightarrow{C_1A_2} = \overrightarrow{C_2B_1} + \overrightarrow{B_2A_1} \iff \overrightarrow{C_2B_1} + \overrightarrow{B_2A_1} - \overrightarrow{C_1A_2} = \vec{0}$$

$$\begin{aligned} \overrightarrow{C_2B_1} + \overrightarrow{B_2A_1} - \overrightarrow{C_1A_2} &= \overrightarrow{C_2B_1} + \overrightarrow{B_2A_1} + \overrightarrow{A_2C_1} \\ &= \overrightarrow{C_2A} + \overrightarrow{AB_1} + \overrightarrow{B_2C} + \overrightarrow{CA_1} + \overrightarrow{A_2B} + \overrightarrow{BC_1} \\ &= \frac{2}{3}\overrightarrow{BA} + \frac{2}{3}\overrightarrow{AC} + \frac{2}{3}\overrightarrow{AC} + \frac{2}{3}\overrightarrow{CB} + \frac{2}{3}\overrightarrow{CB} + \frac{2}{3}\overrightarrow{BA} \\ &= \frac{4}{3}\overrightarrow{BA} + \frac{4}{3}\overrightarrow{AC} + \frac{4}{3}\overrightarrow{CB} = \frac{4}{3}(\overrightarrow{BA} + \overrightarrow{AC} + \overrightarrow{CB}) = \vec{0} \end{aligned}$$

$$3) \quad \overrightarrow{A_1B_1} = \overrightarrow{C_2B_2} \iff \overrightarrow{A_1B_1} - \overrightarrow{C_2B_2} = \vec{0}$$

$$\begin{aligned} \overrightarrow{A_1B_1} - \overrightarrow{C_2B_2} &= \overrightarrow{A_1B_1} + \overrightarrow{B_2C_2} = \overrightarrow{A_1C} + \overrightarrow{CB_1} + \overrightarrow{B_2A} + \overrightarrow{AC_2} \\ &= \frac{2}{3}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{CA} + \frac{1}{3}\overrightarrow{CA} + \frac{2}{3}\overrightarrow{AB} = \frac{2}{3}\overrightarrow{BC} + \frac{2}{3}\overrightarrow{CA} + \frac{2}{3}\overrightarrow{AB} \\ &= \frac{2}{3}(\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}) = \vec{0} \end{aligned}$$