Zadatak 15. Vektori $\vec{a}$ i $\vec{b}$ nisu kolinearni. Za koje realne brojeve α i β su vektori $\vec{c} = \alpha\vec{a} + (\beta - 1)\vec{b}$ i $\vec{d} = (\alpha + 1)\vec{a} + (\beta + 2)\vec{b}$ kolinearni?

Rješenje. $\vec{a}$ i $\vec{b}$ nekolinearni.

$$\vec{c} = \alpha\vec{a} + (\beta - 1)\vec{b}$$
$$\vec{d} = (\alpha + 1)\vec{a} + (\beta + 2)\vec{b}$$
$$\vec{c} = k\vec{d},$$
$$\alpha\vec{a} + (\beta - 1)\vec{b} = k(\alpha + 1)\vec{a} + k(\beta + 2)\vec{b}$$

Izjednačavanjem koeficijenata uz vektore a, odnosno b dobijemo sustav dviju jednadžbi s dvije nepoznanice:

$$\alpha = k(\alpha + 1) \implies k = \frac{\alpha}{\alpha + 1}$$
$$(\beta - 1) = k(\beta + 2) \implies k = \frac{\beta - 1}{\beta + 2}$$

$$\frac{\alpha}{\alpha + 1} = \frac{\beta - 1}{\beta + 2}$$
$$\alpha(\beta + 2) = (\alpha + 1)(\beta - 1)$$
$$\alpha\beta + 2\alpha = \alpha\beta - \alpha + \beta - 1$$
$$3\alpha = \beta - 1 \implies \beta = 3\alpha + 1$$
$$k = \frac{\alpha}{\alpha + 1}, \quad \alpha \neq 0, \ \alpha \neq -1, \ \alpha \in \mathbf{R},$$