Zadatak 3. Točkama B, C, D i E dužina $\overline{AF}$ podijeljena je na pet sukladnih dijelova. Ako je $A(-3,-2)$, $D(3,1)$, odredi koordinate točaka B, C, E i F.

Rješenje.

$A(-3,-2)$
$D(3,1)$

$$|\overrightarrow{AD}| : |\overrightarrow{DB}| = 3:2$$
$$\frac{|\overrightarrow{AD}|}{|\overrightarrow{DB}|} = \frac{3}{2} \implies |\overrightarrow{AD}| = \frac{3}{2}|\overrightarrow{DB}| \implies \lambda = \frac{3}{2}$$
$$x_D = \frac{x_A + \lambda x_F}{1+\lambda}$$
$$3 = \frac{-3+\frac{3}{2}x_F}{1+\frac{3}{2}} / \cdot \frac{5}{2}$$
$$\frac{15}{2} = -3 + \frac{3}{2}x_F / \cdot 2$$
$$15 + 6 = 3x_F$$
$$21 = 3x_F \implies x_F = 7$$
$$y_D = \frac{y_A + \lambda y_F}{1+\lambda}$$
$$1 = \frac{-2+\frac{3}{2}y_F}{1+\frac{3}{2}} / \cdot \frac{5}{2}$$
$$\frac{5}{2} = -2 + \frac{3}{2}y_F / \cdot 2$$
$$5 + 4 = 3y_F$$
$$y_F = 3$$
$$F(7,3)$$

$A(-3,-2)$
$F(7,3)$

$|\overrightarrow{AB}| : |\overrightarrow{BF}| = 1:4$

$$4|\overrightarrow{AB}| = |\overrightarrow{BF}| \implies |\overrightarrow{AB}| = \frac{1}{4}|\overrightarrow{BF}| \implies \lambda = \frac{1}{4}$$

$$x_B = \frac{x_A + \lambda x_F}{1+\lambda} = \frac{-3 + \frac{1}{4}\cdot 7}{1+\frac{1}{4}} = \frac{-\frac{5}{4}}{\frac{5}{4}} = -1$$

$$y_B = \frac{y_A + \lambda y_F}{1+\lambda} = \frac{-2 + \frac{1}{4}\cdot 3}{1+\frac{1}{4}} = \frac{-\frac{5}{4}}{\frac{5}{4}} = -1$$

$B(-1,-1)$

$A(-3,-2)$
$F(7,3)$

$|\overrightarrow{AC}| : |\overrightarrow{CF}| = 2:3$

$$3|\overrightarrow{AC}| = 2|\overrightarrow{CF}| \implies |\overrightarrow{AC}| = \frac{2}{3}|\overrightarrow{CF}| \implies \lambda = \frac{2}{3}$$

$$x_C = \frac{x_A + \lambda x_F}{1+\lambda} = \frac{-3 + \frac{2}{3}\cdot 7}{1+\frac{2}{3}} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1$$

$$y_C = \frac{y_A + \lambda y_F}{1+\lambda} = \frac{-2 + \frac{2}{3}\cdot 3}{1+\frac{2}{3}} = 0$$

$B(1,0)$

$A(-3,-2)$
$F(7,3)$

$|\overrightarrow{AE}| : |\overrightarrow{EF}| = 4:1$

$$|\overrightarrow{AE}| = 4|\overrightarrow{EF}| \implies \lambda = 4$$

$$x_E = \frac{x_A + \lambda x_F}{1+\lambda} = \frac{-3+4\cdot 7}{1+4} = 5$$

$$y_C = \frac{y_A + \lambda y_F}{1+\lambda} = \frac{-2+4\cdot 3}{1+4} = 2$$

$E(5,2)$