Zadatak 18. Dan je trokut ABC; $A(1,3)$, $B(7,0)$ i $C(4,6)$ i točke B_1 i C_1 na njegovim stranicama $\overline{AC}$, odnosno $\overline{AB}$ tako da je $|AB_1| : |B_1C| = 2$ te $|AC_1| : |C_1B| = \frac{1}{2}$. Kolika je površina četverokuta B_1C_1BC?

Rješenje.

$$A(1,3),\ B(7,0),\ C(4,6)$$

$$B_1 \in \overline{AC}$$

$$C_1 \in \overline{AB}$$

$$|AB_1| : |B_1C| = 2 = \lambda_1$$

$$|AC_1| : |C_1B| = \frac{1}{2}\lambda_2$$

$$x_{B_1} = \frac{x_A + \lambda_1 x_C}{1+\lambda_1} = \frac{1+2\cdot 4}{1+2} = \frac{9}{3} = 3$$

$$y_{B_1} = \frac{y_A + \lambda_1 y_C}{1+\lambda_1} = \frac{3+2\cdot 6}{1+2} = \frac{15}{3} = 5$$

$$B_1(3,5)$$

$$x_{C_1} = \frac{x_A + \lambda_2 x_B}{1+\lambda_2} = \frac{1+\frac{1}{2}\cdot 7}{1+\frac{1}{2}} = \frac{\frac{9}{2}}{\frac{3}{2}} = 3$$

$$y_{C_1} = \frac{y_A + \lambda_2 y_B}{1+\lambda_2} = \frac{3+\frac{1}{2}\cdot 0}{1+\frac{1}{2}} = \frac{3}{\frac{3}{2}} = 2$$

$$C_1(3,2)$$

$$\begin{aligned}
P_{B_1C_1BC} &= P_{\triangle B_1C_1B} + P_{\triangle B_1BC} \\
&= \frac{1}{2}[x_{B_1}(y_{C_1} - y_B) + x_{C_1}(y_B - y_{B_1}) + x_B(y_{B_1} - y_{C_1})] \\
&+ \frac{1}{2}[x_{B_1}(y_B - y_C) + x_B(y_C - y_{B_1}) + x_C(y_{B_1} - y_B)] \\
&= \frac{1}{2}[3(2-0) + 3(0-5) + 7(5-2)] \\
&+ \frac{1}{2}[3(0-6) + 7(6-5) + 4(5-0)] \\
&= \frac{1}{2}(6-15+21) + \frac{1}{2}(-18+7+20) = \frac{1}{2}\cdot 12 + \frac{1}{2}\cdot 9 = \frac{21}{2}
\end{aligned}$$