

Zadatak 19. Dan je trokut $\triangle ABC$; $A(-4, -1)$, $B(5, -4)$, $C(2, 5)$. Točke E i F pripadaju stranicama \overline{AB} i \overline{BC} te je $|AE| : |EB| = 1 : 2$ i $|BF| : |FC| = 2 : 1$. Točka G polovište je stranice \overline{AC} . Kolika je površina trokuta $\triangle EFG$?

Rješenje.

$$A(-4, -1), B(5, -4), C(2, 5)$$

$$E \in \overline{AB}$$

$$F \in \overline{BC}$$

$$|AE| : |EB| = 1 : 2 \implies \lambda_1 = \frac{1}{2}$$

$$|BF| : |FC| = 2 : 1 \implies \lambda_2 = 2$$

$$x_E = \frac{x_A + \lambda_1 x_B}{1 + \lambda_1} = \frac{-4 + \frac{1}{2} \cdot 5}{1 + \frac{1}{2}} = \frac{-\frac{3}{2}}{\frac{3}{2}} = -1$$

$$y_E = \frac{y_A + \lambda_1 y_B}{1 + \lambda_1} = \frac{-1 + \frac{1}{2} \cdot (-4)}{1 + \frac{1}{2}} = \frac{-3}{\frac{3}{2}} = -2$$

$$E(-1, -2)$$

$$x_F = \frac{x_B + \lambda_2 x_C}{1 + \lambda_2} = \frac{5 + 2 \cdot 2}{1 + 2} = \frac{9}{3} = 3$$

$$y_F = \frac{y_B + \lambda_2 y_C}{1 + \lambda_2} = \frac{-4 + 2 \cdot 5}{1 + 2} = \frac{6}{3} = 2$$

$$F(3, 2)$$

$$x_G = \frac{x_A + x_C}{2} = \frac{-4 + 2}{2} = -1$$

$$y_G = \frac{y_A + y_C}{2} = \frac{-1 + 5}{2} = 2$$

$$G(-1, 2)$$

$$\begin{aligned} P_{EFG} &= \frac{1}{2} |x_E(y_F - y_G) + x_F(y_G - y_E) + x_G(y_E - y_F)| \\ &= \frac{1}{2} |-1(2 - 2) + 3(2 + 2) - 1(-2 - 2)| \\ &= \frac{1}{2} |12 + 4| = 8 \end{aligned}$$