Zadatak 36. Dan je kvadrat $ABCD$. Točka T_1 težište je trokuta ABC, a točka T_2 težište trokuta ACD. Ako je $\overrightarrow{AT_1} \cdot \overrightarrow{AT_2} = 8$, kolika je površina kvadrata?

Rješenje.

$$|\overrightarrow{AB}| = |\overrightarrow{BC}| \text{ i } \overrightarrow{AB} \cdot \overrightarrow{BC} = 0$$

$$\overrightarrow{AT_1} = \frac{2}{3}\left(\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}\right) = \frac{2}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC}$$

$$\overrightarrow{AT_2} = \frac{2}{3}\left(\overrightarrow{BC} + \frac{1}{2}\overrightarrow{AB}\right) = \frac{1}{3}\overrightarrow{AB} + \frac{2}{3}\overrightarrow{BC}$$

$$\overrightarrow{AT_1} \cdot \overrightarrow{AT_2} = 8$$

$$\frac{2}{9}\overrightarrow{AB}^2 + \frac{4}{9}\overrightarrow{AB} \cdot \overrightarrow{BC} + \frac{1}{9}\overrightarrow{BC} \cdot \overrightarrow{AB} + \frac{2}{9}\overrightarrow{BC}^2 = 8$$

$$\frac{2}{9}|\overrightarrow{AB}|^2 + \frac{2}{9}|\overrightarrow{BC}|^2 + \frac{5}{9}\overrightarrow{AB} \cdot \overrightarrow{BC} = 8$$

$$\frac{4}{9}|\overrightarrow{AB}|^2 = 8 / \cdot \frac{9}{4}$$

$$|\overrightarrow{AB}|^2 = 18$$

$$P = 18$$