

Zadatak 39. Točke $A(-1, -2)$ i $C(5, 0)$ dva su suprotna vrha kvadrata $ABCD$. Odredi koordinate vrhova B i D tog kvadrata.

Rješenje.

Točka P polovište je dijagonale \overline{AC} kvadrata.

$$P\left(\frac{x_A + x_C}{2}, \frac{y_A + y_C}{2}\right) = P\left(\frac{-1 + 5}{2}, \frac{-2 + 0}{2}\right) = P(2, -1).$$

$$\overrightarrow{AP} \perp \overrightarrow{PD}, \overrightarrow{AP} \perp \overrightarrow{BP} \text{ i } \overrightarrow{PD} = \overrightarrow{BP}$$

$$\overrightarrow{AP} = (2 + 1)\vec{i} + (-1 + 2)\vec{j} = 3\vec{i} + \vec{j}$$

$$\overrightarrow{BP} = (2 - x_B)\vec{i} + (-1 - y_B)\vec{j}$$

$$\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$$

$$3(2 - x_B) - 1 - y_B = 0 \implies 6 - 3x_B - 1 - y_B = 0 \implies 3x_B + y_B = 5$$

$$|\overrightarrow{AP}| = \sqrt{9 + 1} = \sqrt{10} = |\overrightarrow{BP}|$$

$$\sqrt{(2 - x_B)^2 + (-1 - y_B)^2} = \sqrt{10}/^2$$

$$4 - 4x_B + x_B^2 + 1 + 2y_B + y_B^2 = 10$$

$$5 - 4x_B + x_B^2 + 2(5 - 3x_B) + (5 - 3x_B)^2 = 10$$

$$-4x_B + x_B^2 + 10 - 6x_B + 25 - 30x_B + 9x_B^2 = 5$$

$$-40x_B + 10x_B^2 + 30 = 0 / : 10$$

$$x_B^2 - 4x_B + 3 = 0$$

$$x_{B_{1,2}} = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2}$$

$$x_{B_1} = 3 \quad x_{B_2} = 1$$

$$y_{B_{1,2}} = 5 - 3x_B$$

$$y_{B_1} = -4 \quad y_{B_2} = 2$$

$$B(3, -4) \quad D(1, 2)$$