Zadatak 40. Točke $A(-1,-1)$ i $B(3,2)$ uzastopni su vrhovi kvadrata $ABCD$. Odredi ostala dva vrha tog kvadrata.

Rješenje.

$$\overrightarrow{AB} = (3+1)\vec{\imath} + (2+1)\vec{\jmath} = 4\vec{\imath} + 3\vec{\jmath}$$
$$|\overrightarrow{AB}| = \sqrt{16+9} = 5 = |\overrightarrow{AD}$$
$$\overrightarrow{AD} = (x_D+1)\vec{\imath} + (y_D+1)\vec{\jmath}$$
$$\overrightarrow{AB} \perp \overrightarrow{AD} \implies 4(x_D+1) + 3(y_D+1) = 0$$

$$4x_D + 4 + 3y_D + 3 = 0$$
$$(x_D+1)^2 + (y_D+1)^2 = 25$$

$$4x_D + 3y_D = -7$$
$$x_D^2 + 2x_D + 1 + y_D^2 + 2y_D + 1 = 25$$

$$x_D = -\frac{3}{4}y_D - \frac{7}{4}$$
$$x_D^2 + 2x_D + y_D^2 + 2y_D = 23$$

$$\left(-\frac{3}{4}y_D - \frac{7}{4}\right)^2 + 2\left(-\frac{3}{4}y_D - \frac{7}{4}\right) + y_D^2 + 2y_D = 23$$
$$\frac{9}{16}y_D^2 + \frac{21}{8}y_D + \frac{49}{16} - \frac{3}{2}y_D - \frac{7}{2} + y_D^2 + 2y_D = 23 / \cdot 16$$
$$9y_D^2 + 42y_D + 49 - 24y_D - 56 + 16y_D^2 + 32y_D = 368$$
$$25y_D^2 + 50y_D = 375 / : 25$$
$$y_D^2 + 2y_D - 15 = 0$$
$$y_{D_{1,2}} = \frac{-2 \pm \sqrt{4+60}}{2}$$
$$y_{D_{1,2}} = \frac{-2 \pm 8}{2}$$
$$y_{D_{1,2}} = -1 \pm 4$$
$$x_{D_{1,2}} = -\frac{3}{4}(-1 \pm 4) - \frac{7}{4}$$
$$D_1(-4,3) \quad D_2(2,-5)$$
$$\overrightarrow{AD_1} = \overrightarrow{BC_1}$$
$$(-4+1)\vec{\imath} + (3+1)\vec{\jmath} = (x_{C_1}-3)\vec{\imath} + (y_{C_1}-2)\vec{\jmath}$$
$$x_{C_1} - 3 = -3 \implies x_{C_1} = 0$$
$$y_{C_1} - 2 = 4 \implies y_{C_1} = 6$$
$$C_1(0,6)$$

$$\overrightarrow{AD_2} = \overrightarrow{BC_2}$$
$$(2+1)\vec{\imath} + (-5+1)\vec{\jmath} = (x_{C_2} - 3)\vec{\imath} + (y_{C_2} - 2)\vec{\jmath}$$
$$x_{C_2} - 3 = 3 \implies x_{C_1} = 6$$
$$y_{C_2} - 2 = -4 \implies y_{C_1} = -2$$
$$C_1(6, -2)$$