

Zadatak 11. Izravnim računom provjeri jednakosti:

$$1) \binom{8}{3} + \binom{8}{4} = \binom{9}{4};$$

$$2) \binom{10}{4} + \binom{10}{5} = \binom{11}{5};$$

$$3) \binom{15}{7} + \binom{15}{8} = \binom{16}{8};$$

$$4) \binom{n+1}{n-1} + \binom{n+1}{n} = \binom{n+2}{n}.$$

Rješenje.

$$1) \binom{8}{3} + \binom{8}{4} = \frac{6 \cdot 7 \cdot 8}{2 \cdot 3} + \frac{5 \cdot 6 \cdot 7 \cdot 8}{2 \cdot 3 \cdot 4} = 56 + 70 = 126,$$

$$\binom{9}{4} = \frac{9!}{4!5!} = \frac{6 \cdot 7 \cdot 8 \cdot 9}{2 \cdot 3 \cdot 4} = 63 \cdot 2 = 126;$$

$$2) \binom{10}{4} + \binom{10}{5} = \frac{7 \cdot 8 \cdot 9 \cdot 10}{2 \cdot 3 \cdot 4} + \frac{6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{2 \cdot 3 \cdot 4 \cdot 5} = 210 + 252 = 462,$$

$$\binom{11}{5} = \frac{11!}{5!6!} = \frac{7 \cdot 8 \cdot 9 \cdot 10 \cdot 11}{2 \cdot 3 \cdot 4 \cdot 5} = 462;$$

$$5) \binom{5}{0} + \binom{5}{2} + \binom{5}{4} = 6 + 10 = 16,$$

$$\binom{5}{1} + \binom{5}{3} + \binom{5}{5} = 5 + \frac{4 \cdot 5}{2} + 1 = 6 + 10 = 16;$$

$$6) \binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6} = 2 + \frac{5 \cdot 6}{2} + \frac{5 \cdot 6}{2} = 32,$$

$$\binom{6}{1} + \binom{6}{3} + \binom{6}{5} = 12 + \frac{4 \cdot 5 \cdot 6}{2 \cdot 3} = 32.$$