Zadatak 12. Odredi prirodni broj n tako da vrijede jednakosti:

1) $\binom{n}{5}=\binom{n}{3}$; **2)** $2\binom{n}{4}=\binom{n+1}{4}$;

3) $7\binom{n}{4}=\binom{n+2}{4}$; **4)** $5\binom{n}{3}=\binom{n+2}{4}$;

5) $3\binom{2n}{n-1}=5\binom{2n-1}{n}$;

6) $17\binom{2n-1}{n}=9\binom{2n}{n-1}$.

Rješenje. **1)**

$$\binom{n}{5}=\binom{n}{3}$$
$$\frac{n(n-1)(n-2)(n-3)(n-4)}{5!}=\frac{n(n-1)(n-2)}{3!}$$
$$\frac{(n-3)(n-4)}{5\cdot 4}=1$$
$$n^2-7n+12=20$$
$$n^2-7n-8=0$$
$$(n-8)(n+1)=0$$
$$n=8;$$

2)

$$2\binom{n}{4}=\binom{n+1}{4}$$
$$2\frac{n(n-1)(n-2)(n-3)}{4!}=\frac{(n+1)n(n-1)(n-2)}{4!}$$
$$2(n-3)=n+1$$
$$2n-6=n+1$$
$$n=7;$$

3)

$$7\binom{n}{4}=\binom{n+2}{4}$$
$$7\frac{n(n-1)(n-2)(n-3)}{4!}=\frac{(n+2)(n+1)n(n-1)}{4!}$$
$$7(n-2)(n-3)=(n+1)(n+2)$$
$$7n^2-35n+42=n^2+3n+2$$
$$3n^2-19n+20=0$$
$$n_{1,2}=\frac{19\pm\sqrt{19^2-4\cdot 3\cdot 20}}{6}=\frac{19\pm 11}{6}$$
$$n=5;$$

4)

$$5\binom{n}{3}=\binom{n+2}{4}$$
$$5\frac{n(n-1)(n-2)}{6}=\frac{(n+2)(n+1)n(n-1)}{24}$$
$$20(n-2)=(n+2)(n+1)$$
$$20n-40=n^2+3n+2$$
$$n^2-17n+42=0$$
$$n_{1,2}=\frac{17\pm\sqrt{17^2-4\cdot 42}}{2}=\frac{17\pm 11}{2}$$
$$n_1=14,\quad n_2=3;$$

5)

$$3\binom{2n}{n-1}=5\binom{2n-1}{n}$$
$$3\frac{(2n)(2n-1)\cdots(n+2)}{(n-1)!}=5\frac{(2n-1)(2n-2)\cdots n}{n!}$$
$$3\cdot 2n=5\cdot(n+1)$$
$$6n=5n+5$$
$$n=5;$$

6)

$$17\binom{2n-1}{n}=9\binom{2n}{n-1}$$
$$17\frac{(2n-1)(2n-2)\cdots(n+1)n}{n!}=9\frac{(2n)(2n-1)\cdots(n+3)(n+2)}{(n-1)!}$$
$$17\cdot(n+1)=9\cdot 2n$$
$$17n+17=18n$$
$$n=17.$$