



**Zadatak 15.** Prikaži pomoću binomne formule:

1)  $(x - 1)^4$ ;

2)  $(2x + 1)^5$ ;

3)  $(2x + 1)^6$ ;

4)  $\left(x + \frac{1}{x}\right)^4$ ;

5)  $\left(x - \frac{1}{x}\right)^5$ ;

6)  $(1 + y^2)^4$ ;

7)  $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^6$ ;

8)  $(\sqrt{3} - \sqrt{2})^6$ .

**Rješenje.**

U prikazima koristimo koeficijente Pascalovog trokuta

1	1	1	1
1	1	3	1
1	4	6	4
1	5	10	10
1	6	15	15
1	7	21	21
1	28	56	56
8	56	70	70
28	35	35	35
1	21	15	15
1	7	6	5
1	1	1	1

1)

$$\begin{aligned}(x - 1)^4 &= x^4 + \binom{4}{1}x^3(-1) + \binom{4}{2}x^2(-1)^2 + \binom{4}{3}x(-1)^3 + (-1)^4 \\ &= x^4 - 4x^3 + 6x^2 - 4x + 1\end{aligned}$$

2)

$$\begin{aligned}(2x + 1)^5 &= (2x)^5 + \binom{5}{1}(2x)^4 + \binom{5}{2}(2x)^3 + \binom{5}{3}(2x)^2 + \binom{5}{4}(2x) + 1 \\ &= 32x^5 + 80x^4 + 80x^3 + 40x^2 + 10x + 1\end{aligned}$$

3)

$$\begin{aligned}(2x + 1)^6 &= (2x)^6 + \binom{6}{1}(2x)^5 + \binom{6}{2}(2x)^4 + \binom{6}{3}(2x)^3 + \binom{6}{4}(2x)^2 + \binom{6}{5}(2x) + 1 \\ &= 64x^6 + 192x^5 + 192x^4 + 160x^3 + 60x^2 + 12x + 1\end{aligned}$$

4)

$$\begin{aligned}\left(x + \frac{1}{x}\right)^4 &= x^4 + \binom{4}{1}x^3\frac{1}{x} + \binom{4}{2}x^2\frac{1}{x^2} + \binom{4}{3}x\frac{1}{x^3} + \frac{1}{x^4} \\ &= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}\end{aligned}$$

5)

$$\begin{aligned}\left(x - \frac{1}{x}\right)^5 &= x^5 - \binom{5}{1}x^4\frac{1}{x} + \binom{5}{2}x^3\frac{1}{x^2} - \binom{5}{3}x^2\frac{1}{x^3} + \binom{5}{4}x\frac{1}{x^4} - \frac{1}{x^5} \\ &= x^5 - 5x^3 + 10x - x^2 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}\end{aligned}$$

**6)**

$$\begin{aligned}(1+y^2)^4 &= 1 + \binom{4}{1}(y^2) + \binom{4}{2}(y^2)^2 + \binom{4}{1}(y^2)^3 + (y^2)^4 \\&= 1 + 4y^2 + 6y^4 + 4y^6 + y^8\end{aligned}$$

**7)**

$$\begin{aligned}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^6 &= (\sqrt{x})^6 - \binom{6}{1}(\sqrt{x})^5 \frac{1}{\sqrt{x}} + \binom{6}{2}(\sqrt{x})^4 \frac{1}{(\sqrt{x})^2} \\&\quad - \binom{6}{3}(\sqrt{x})^3 \frac{1}{(\sqrt{x})^3} + \binom{6}{2}(\sqrt{x})^2 \frac{1}{(\sqrt{x})^4} \\&\quad - \binom{6}{1}(\sqrt{x}) \frac{1}{(\sqrt{x})^5} + \frac{1}{(\sqrt{x})^6} \\&= x^3 - 6x^2 + 15x - 20 + \frac{15}{x} - \frac{6}{x^2} + \frac{1}{x^3}\end{aligned}$$