Zadatak 16. Izračunaj:

1) $(3x+1)^4+(3x-1)^4$;

2) $(x+1)^6+(x-1)^6$.

Rješenje. Razvijmo po binomnoj formuli i koristimo koeficijente Pascalovog trokuta.

1) $(3x+1)^4+(3x-1)^4$

$$\begin{aligned}
&= \binom{4}{0}(3x)^4+\binom{4}{1}(3x)^3+\binom{4}{2}(3x)^2+\binom{4}{3}(3x)^1+\binom{4}{4}(3x)^0\\
&\quad+\binom{4}{0}(3x)^4(-1)^0+\binom{4}{1}(3x)^3(-1)^1+\binom{4}{2}(3x)^2(-1)^2\\
&\quad+\binom{4}{3}(3x)^1(-1)^3+\binom{4}{4}(3x)^0(-1)^4\\
&= 81x^4+4\cdot 27x^3+6\cdot 9x^2+4\cdot 3x+1+81x^4-4\cdot 27x^3+6\cdot 9x^2-4\cdot 3x+1\\
&= 162x^4+108x^2+2;
\end{aligned}$$

2) $(x+1)^6+(x-1)^6$

$$\begin{aligned}
&= \binom{6}{0}x^6+\binom{6}{1}x^5+\binom{6}{2}x^4+\binom{6}{3}x^3+\binom{6}{4}x^2+\binom{6}{5}x^1+\binom{6}{6}x^0\\
&\quad+\binom{6}{0}x^6-\binom{6}{1}x^5+\binom{6}{2}x^4-\binom{6}{3}x^3+\binom{6}{4}x^2-\binom{6}{5}x^1+\binom{6}{6}x^0\\
&= x^6+6x^5+15x^4+20x^3+15x^2+6x+1+x^6-6x^5+15x^4-20x^3\\
&\quad+15x^2-6x+1=2x^6+30x^4+30x^2+2\ ;
\end{aligned}$$