

Zadatak 33. Koji je broj veći:

$$1) A = \frac{54\,311\,215}{54\,311\,216} \text{ ili } B = \frac{45\,311\,521}{45\,311\,522};$$

$$2) A = \frac{1\,234\,512\,345}{4\,567\,845\,678} \text{ ili } B = \frac{1\,234\,512\,346}{4\,567\,845\,679}.$$

Rješenje.

1) Usporedimo brojeve $1 - A$ i $1 - B$.

$$1 - A = \frac{54\,311\,216}{54\,311\,216} - \frac{54\,311\,215}{54\,311\,216} = \frac{1}{54\,311\,216}$$

$$1 - B = \frac{45\,311\,522}{45\,311\,522} - \frac{45\,311\,521}{45\,311\,522} = \frac{1}{45\,311\,522}$$

$$1 - B > 1 - A \implies A > B.$$

2) Ako je $A = \frac{a}{b}$, onda je $B = \frac{a+1}{b+1}$. Tada je

$$A - B = \frac{a}{b} - \frac{a+1}{b+1} = \frac{a(b+1) - b(a+1)}{b(b+1)} = \frac{a-b}{b(b+1)}.$$

U našem je slučaju $a = 1\,234\,512\,345$, $b = 4\,567\,845\,678$ te je

$$A - B = \frac{1\,234\,512\,345 - 4\,567\,845\,678}{4\,567\,845\,678(4\,567\,845\,678 + 1)} < 0$$

iz čega slijedi $A < B$.