Zadatak 19. Izračunaj:

1) $\sqrt[3]{2+\sqrt{3}}\cdot\sqrt{7-4\sqrt{3}}$;

2) $\sqrt{5\sqrt{2}-7}\cdot\sqrt[3]{7+5\sqrt{2}}$;

3) $\left[16^{-0.25}-(2\sqrt{2})^{\frac{1}{3}}\right]\left[16^{-0.25}+(2\sqrt{2})^{\frac{1}{3}}\right]$;

4) $\left[9^{-\frac{1}{2}}+(3\sqrt{3})^{-\frac{2}{3}}\right]\left[9^{-\frac{1}{2}}-(3\sqrt{3})^{-\frac{2}{3}}\right]$.

Rješenje.

1) $\sqrt[3]{2+\sqrt{3}}\cdot\sqrt{7-4\sqrt{3}}=\sqrt[3]{2+\sqrt{3}}\cdot\sqrt{(2-\sqrt{3})^2}$

$=\sqrt[3]{(2+\sqrt{3})\cdot(2-\sqrt{3})}=\sqrt[3]{(2+\sqrt{3})(2-\sqrt{3})(2+\sqrt{3})^2}$

$=\sqrt[3]{(4-3)(2-\sqrt{3})^2}=\sqrt[3]{7-4\sqrt{3}}$;

2) $\sqrt{5\sqrt{2}-7}\cdot\sqrt[3]{7+5\sqrt{2}}=\sqrt[6]{(5\sqrt{2}-7)^3(7+5\sqrt{2})^2}$

$=\sqrt[6]{(5\sqrt{2}-7)[(5\sqrt{2}-7)(5\sqrt{2}+7)]^2}=\sqrt[6]{(5\sqrt{2}-7)(50-49)^2}$

$=\sqrt[6]{5\sqrt{2}-7}$;

3) $\left[16^{-0.25}-(2\sqrt{2})^{\frac{1}{3}}\right]\left[16^{-0.25}+(2\sqrt{2})^{\frac{1}{3}}\right]$

$=\left[(2^4)^{-\frac{1}{4}}-\sqrt[3]{2\sqrt{2}}\right]\left[(2^4)^{-\frac{1}{4}}+\sqrt[3]{2\sqrt{2}}\right]$

$=\left(\frac{1}{2}-\sqrt[3]{2\sqrt{2}}\right)\left(\frac{1}{2}+\sqrt[3]{2\sqrt{2}}\right)=\frac{1}{4}-\sqrt[3]{(2\sqrt{2})^2}$

$=\frac{1}{4}-\sqrt[3]{8}=\frac{1}{4}-2=-\frac{7}{4}$;

4) $\left[9^{-\frac{1}{2}}+(3\sqrt{3})^{-\frac{2}{3}}\right]\left[9^{-\frac{1}{2}}-(3\sqrt{3})^{-\frac{2}{3}}\right]$

$=\left[(3^2)^{-\frac{1}{2}}+\sqrt[3]{(3\sqrt{3})^{-2}}\right]\left[(3^2)^{-\frac{1}{2}}-\sqrt[3]{(3\sqrt{3})^{-2}}\right]$

$=\left(\frac{1}{3}+\sqrt[3]{\frac{1}{27}}\right)\left(\frac{1}{3}-\sqrt[3]{\frac{1}{27}}\right)=\left(\frac{1}{3}+\frac{1}{3}\right)\left(\frac{1}{3}-\frac{1}{3}\right)=0$;