Zadatak 5. Neka je $w = \dfrac{z-1}{z+1}$, $z \neq \pm 1$. Dokaži da je $\operatorname{Re} w = 0$ ako i samo ako je $|z| = 1$.

Rješenje.

$$z = a + bi$$

$$w = \frac{(a-1)+bi}{(a+1)+bi} \cdot \frac{(a+1)-bi}{(a+1)-bi} = \frac{(a^2-1)+(a+1)bi-(a-1)bi+b^2}{(a+1)^2+b^2}$$

$$= \frac{a^2-1+b^2+(ab+b-ab+b)i}{(a+1)^2+b^2} = \frac{a^2+b^2-1}{(a+1)^2+b^2} + \frac{2b}{(a+1)^2+b^2}i$$

$$\operatorname{Re} w = \frac{a^2+b^2-1}{(a+1)^2+b^2} = 0 \iff a^2+b^2-1=0 \iff a^2+b^2=1$$

$$\iff |z|^2 = 1 \iff |z| = 1.$$