Zadatak 7. Odredi sve kompleksne brojeve z za koje vrijedi $\overline{z}=z^3$.

Rješenje. $\overline{z}=z^3$, $z\in\mathbf{C}$;

$$a-bi=(a+bi)^3$$
$$a-bi=a^3+3a^2bi-3ab^2-b^3i$$
$$(a^3-3ab^2-a)+(3a^2b-b^3+b)i=0$$

$$\begin{aligned}&a^3-3ab^2-a=0 && 3a^2b-b^3+b=0\\ &a(a^2-3b^2-1)=0 && b(3a^2-b^2+1)=0\\ &a=0 \text{ ili } a^2-3b^2=1 \quad \text{i} && b=0 \text{ ili } 3a^2-b^2=-1\end{aligned}$$

1° $a=0,\ b=0 \implies \boxed{z=0}$

2° $a=0,\ 3a^2-b^2=-1 \implies -b^2=-1 \implies b^2=1 \implies b=\pm1 \implies \boxed{z=\pm i}$

3° $a^2-3b^2=1,\ b=0 \implies a^2=1 \implies a=\pm1 \implies \boxed{z=\pm1}$

4° $\left.\begin{aligned}a^2-3b^2&=1\\ 3a^2-b^2&=-1\end{aligned}\right\}$ presjek dvije hiperbole

$$b^2=3a^2+1$$
$$a^2-3(3a^2+1)=1$$
$$a^2-9a^2-3=1$$
$$-8a^2=4$$
$$a^2=-\frac{1}{2} \implies a\notin\mathbf{R} \implies \text{ove hiperbole se ne sijeku.}$$