Zadatak 12. Za točke iz prethodnog zadatka odredi Kartezijeve koordinate.

Rješenje.

1) $z_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\,;$

2) $z_2 = 2\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) = 2\left(\cos\left(\frac{\pi}{2} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)\right)$

$= 2\left(\sin\frac{3\pi}{4} + i\cos\frac{3\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = \sqrt{2} - \sqrt{2}i\,;$

3) $z_3 = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 3i\,;$

4) $z_4 = 2\,(\cos\pi + i\sin\pi) = -2\,;$

5) $z_5 = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) = \cos\left(\pi - \frac{3\pi}{2}\right) + i\sin\left(\pi - \frac{3\pi}{2}\right)$

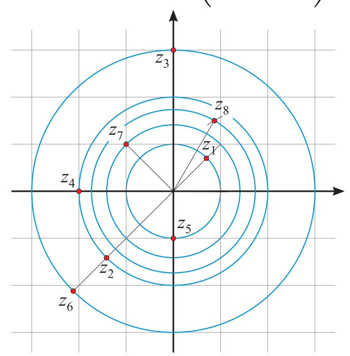
$= -\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i\,;$

6) $z_6 = 3\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = 3\left(\cos\left(\pi + \frac{\pi}{4}\right) + i\sin\left(\pi + \frac{\pi}{4}\right)\right)$

$= 3\left(-\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right) = -3\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i\,;$

7) $z_7 = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -1 + i\,;$

8) $z_8 = \sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = \sqrt{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \frac{\sqrt{3}}{2} + \frac{3}{2}i\,.$



$x = r\cos\varphi\,,\ y = r\sin\varphi$

1) $\left(1, \frac{\pi}{4}\right) \Longrightarrow \left.\begin{array}{l} x = 1\cdot\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \\ y = 1\cdot\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \end{array}\right\} \Longrightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right).$

2) $\left(2, -\frac{\pi}{4}\right) \Longrightarrow \left.\begin{array}{l} x = 2\cdot\cos\left(-\frac{\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2} \\ y = 2\cdot\sin\left(-\frac{\pi}{4}\right) = -2\cdot\frac{\sqrt{2}}{2} = -\sqrt{2} \end{array}\right\} \Longrightarrow (\sqrt{2}, -\sqrt{2})\,.$

$$\textbf{3)}\ \left(3, \frac{\pi}{2}\right) \Longrightarrow \left.\begin{array}{l} x = 3 \cdot \cos \frac{\pi}{2} = 3 \cdot 0 = 0 \\ y = 3 \cdot \sin \frac{\pi}{2} = 3 \cdot 1 = 3 \end{array}\right\} \Longrightarrow (0, 3).$$

$$\textbf{4)}\ (2, \pi) \Longrightarrow \left.\begin{array}{l} x = 2 \cdot \cos \pi = 2 \cdot (-1) = -2 \\ y = 2 \cdot \sin \pi = 2 \cdot 0 = 0 \end{array}\right\} \Longrightarrow (-2, 0).$$

$$\textbf{5)}\ \left(1, -\frac{\pi}{2}\right) \Longrightarrow \left.\begin{array}{l} x = 1 \cdot \cos\left(-\frac{\pi}{2}\right) = 1 \cdot 0 = 0 \\ y = 1 \cdot \sin\left(-\frac{\pi}{2}\right) = 1 \cdot (-1) = -1 \end{array}\right\} \Longrightarrow (0, -1).$$

$$\textbf{6)}\ \left(3, \frac{5\pi}{4}\right) \Longrightarrow \left.\begin{array}{l} x=3\cdot \cos \frac{5\pi}{4}=3\cdot\left(-\frac{\sqrt{2}}{2}\right)=-\frac{3\sqrt{2}}{2} \\ y=3\cdot \sin \frac{5\pi}{4}=3\cdot\left(-\frac{\sqrt{2}}{2}\right)=-\frac{3\sqrt{2}}{2} \end{array}\right\} \Longrightarrow \left(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right).$$

$$\textbf{7)}\ \left(\sqrt{2}, \frac{3\pi}{4}\right) \Longrightarrow \left.\begin{array}{l} x = \sqrt{2} \cdot \cos \frac{3\pi}{4} = \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -1 \\ y = \sqrt{2} \cdot \sin \frac{3\pi}{4} = \sqrt{2}\frac{\sqrt{2}}{2} = 1 \end{array}\right\} \Longrightarrow (-1, 1).$$

$$\textbf{8)}\ \left(\sqrt{3}, \frac{\pi}{3}\right) \Longrightarrow \left.\begin{array}{l} x = \sqrt{3} \cdot \cos \frac{\pi}{3} = \sqrt{3}\frac{1}{2} = \frac{\sqrt{3}}{2} \\ y = \sqrt{3} \cdot \sin \frac{\pi}{3} = \sqrt{3}\frac{\sqrt{3}}{2} = \frac{3}{2} \end{array}\right\} \Longrightarrow \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right).$$