

Zadatak 17. Zapiši u trigonometrijskom obliku sljedeće kompleksne brojeve:

$$1) z = 3 \cos \frac{11\pi}{4} - 3i \sin \frac{5\pi}{4};$$

$$2) z = -\cos \frac{\pi}{11} + i \sin \frac{\pi}{11};$$

$$3) z = 3 \left(\sin \frac{\pi}{12} - i \cos \frac{\pi}{12} \right);$$

$$4) z = -\sqrt{2} \cos \frac{5\pi}{4} - i\sqrt{2} \sin \frac{11\pi}{4};$$

$$5) z = 1 + \cos \frac{10\pi}{9} + i \sin \frac{10\pi}{9};$$

$$6) z = 1 - \cos \frac{5\pi}{3} - i \sin \frac{5\pi}{3}.$$

Rješenje.

$$1) z = 3 \cos \frac{11\pi}{4} - 3i \sin \frac{5\pi}{4} = 3 \cos \frac{3\pi}{4} + 3i \sin \frac{3\pi}{4} = 3 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right);$$

$$2) z = -\cos \frac{\pi}{11} + i \sin \frac{\pi}{11} = \cos \frac{10\pi}{11} + i \sin \frac{10\pi}{11};$$

$$3) z = 3 \left(\sin \frac{\pi}{12} - i \cos \frac{\pi}{12} \right) = 3 \left(\cos \frac{5\pi}{12} - i \sin \frac{5\pi}{12} \right) = 3 \left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12} \right);$$

$$4) z = -\sqrt{2} \cos \frac{5\pi}{4} - i\sqrt{2} \sin \frac{11\pi}{4} = \sqrt{2} \left(-\cos \frac{5\pi}{4} - i \sin \frac{3\pi}{4} \right) = \sqrt{2} \left(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right) = \sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right);$$

$$5) z = 1 + \cos \frac{10\pi}{9} + i \sin \frac{10\pi}{9} = 1 + \cos^2 \frac{5\pi}{9} - \sin^2 \frac{5\pi}{9} + 2i \sin \frac{5\pi}{9} \cos \frac{5\pi}{9} = 2 \cos^2 \frac{5\pi}{9} + 2i \sin \frac{5\pi}{9} \cos \frac{5\pi}{9} = 2 \cos \frac{5\pi}{9} \left(\cos \frac{5\pi}{9} + i \sin \frac{5\pi}{9} \right);$$

$$6) z = 1 - \cos \frac{5\pi}{3} - i \sin \frac{5\pi}{3} = 1 - \cos^2 \frac{5\pi}{6} + \sin^2 \frac{5\pi}{6} - 2i \sin \frac{5\pi}{6} \cos \frac{5\pi}{6} = 2 \sin^2 \frac{5\pi}{6} - 2i \sin \frac{5\pi}{6} \cos \frac{5\pi}{6} = 2 \sin \frac{5\pi}{6} \left(\sin \frac{5\pi}{6} - i \cos \frac{5\pi}{6} \right) = 2 \cdot \frac{1}{2} \left(\sin \frac{\pi}{6} + i \cos \frac{\pi}{6} \right) = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}.$$