Zadatak 20. Zapiši u trigonometrijskom obliku brojeve:

1) $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$;

2) $z = \dfrac{1+i\sqrt{3}}{2i\left(\cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}\right)}$;

3) $z = \dfrac{\sqrt{3}-i}{i\left(\cos\frac{7\pi}{6} - i\sin\frac{7\pi}{6}\right)}$;

4) $z = \dfrac{i-1}{i\left(1-\cos\frac{2\pi}{5}\right) + \sin\frac{2\pi}{5}}$.

Rješenje. **1)** $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = \dfrac{\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = \sqrt{2}\left(\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right) = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$;

2) $z = \dfrac{1+i\sqrt{3}}{2i\left(\cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}\right)} = \dfrac{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{2\left(\sin\frac{5\pi}{3} + i\cos\frac{5\pi}{3}\right)} = \dfrac{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}{-\sin\frac{\pi}{3} + i\cos\frac{\pi}{3}} = \dfrac{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}{-\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}} = \dfrac{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}{\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}} = \cos\left(\frac{\pi}{3} - \frac{5\pi}{6}\right) + i\sin\left(\frac{\pi}{3} - \frac{5\pi}{6}\right) = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$;

3) $z = \dfrac{\sqrt{3}-i}{i\left(\cos\frac{7\pi}{6} - i\sin\frac{7\pi}{6}\right)} = \dfrac{2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}{\sin\frac{7\pi}{6} + i\cos\frac{7\pi}{6}} = \dfrac{2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}{\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}} = 2\left(\cos\left(\frac{11\pi}{6} - \frac{4\pi}{3}\right) + i\sin\left(\frac{11\pi}{6} - \frac{4\pi}{3}\right)\right) = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$;

4) $z = \dfrac{i-1}{i\left(1-\cos\frac{2\pi}{5}\right) + \sin\frac{2\pi}{5}} = \dfrac{\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}{2i\sin^2\frac{\pi}{5} + 2\sin\frac{\pi}{5}\cos\frac{\pi}{5}} = \dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{\sin\frac{\pi}{5}}\,\dfrac{\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}}{\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}} = \dfrac{\sqrt{2}}{2\sin\frac{\pi}{5}}\left(\cos\left(\frac{3\pi}{4} - \frac{\pi}{5}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{5}\right)\right) = \dfrac{\sqrt{2}}{2\sin\frac{\pi}{5}}\left(\cos\frac{11\pi}{20} + i\sin\frac{11\pi}{20}\right)$.