

Zadatak 3. Izračunaj sljedeće potencije:

- 1) $\left[2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]^3$; 2) $\left[2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]^4$;
 3) $\left[\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^4$; 4) $\left[\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^6$;
 5) $\left[4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]^{-2}$; 6) $\left[3\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)\right]^{-4}$.

Rješenje.

- 1) $\left[2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]^3 = 2^3(\cos \pi + i \sin \pi) = -8$;
 2) $\left[2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]^4 = 2^4\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right) = 2^4\left(\cos\left(\pi + \frac{\pi}{3}\right) + i \sin\left(\pi + \frac{\pi}{3}\right)\right) = 2^4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -8 - 8i\sqrt{3}$;
 3) $\left[\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^4 = 4(\cos \pi + i \sin \pi) = -4$;
 4) $\left[\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^6 = 2^3\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) = -8i$;
 5) $\left[4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]^{-2} = 4^{-2}\left(\cos \frac{-2\pi}{3} + i \sin \frac{-2\pi}{3}\right)$
 $= 4^{-2}\left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3}\right) = -\frac{1}{32} - \frac{\sqrt{3}}{32}i$;
 6) $\left[3\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)\right]^{-4} = 3^{-4}\left(\cos \frac{-\pi}{2} + i \sin \frac{-\pi}{2}\right) = 3^{-4}\left(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}\right) = -\frac{i}{81}$.