

Zadatak 4. Koliko je $z_1^{12} : z_2^5$ ako je

$$z_1 = \sqrt{2} \left(-\cos \frac{\pi}{4} + i \sin \frac{3\pi}{4} \right),$$

$$z_2 = 2\sqrt{2} \sin \frac{5\pi}{16} - i2\sqrt{2} \cos \frac{5\pi}{16}?$$

Rješenje.

$$\begin{aligned} z_1 &= \sqrt{2} \left(-\cos \frac{\pi}{4} + i \sin \frac{3\pi}{4} \right) = \sqrt{2} \left(\cos \left(\pi - \frac{\pi}{4} \right) + i \sin \frac{3\pi}{4} \right) = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \\ z_2 &= 2\sqrt{2} \sin \frac{5\pi}{16} - i2\sqrt{2} \cos \frac{5\pi}{16} = 2\sqrt{2} \left(\cos \frac{3\pi}{16} - i \sin \frac{3\pi}{16} \right) \\ &= 2\sqrt{2} \left(\cos \frac{29\pi}{16} + i \sin \frac{29\pi}{16} \right) \\ \left[\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \right]^{12} &: \left[2\sqrt{2} \left(\cos \frac{29\pi}{16} + i \sin \frac{29\pi}{16} \right) \right]^5 \\ &= \left[2^6 (\cos \pi + i \sin \pi) \right] : \left[2^{\frac{15}{2}} \left(\cos \frac{17\pi}{16} + i \sin \frac{17\pi}{16} \right) \right] \\ &= 2^{-\frac{3}{2}} \left[\cos \left(-\frac{\pi}{16} \right) + i \sin \left(-\frac{\pi}{16} \right) \right] = \frac{\sqrt{2}}{4} \left(\cos \frac{31\pi}{16} + i \sin \frac{31\pi}{16} \right). \end{aligned}$$