Zadatak 8. Ako je $z_1=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$, $z_2=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$, izračunaj $z_1^n+z_2^n$, $n\in\mathbf{Z}$.

Rješenje. $z_1=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$, $z_2=\bar{z}_1$, $n\in\mathbf{Z}$;

$$z_1^n=\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^n=\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)^n=\cos\frac{2n\pi}{3}+i\sin\frac{2n\pi}{3}$$

$$z_2^n=\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^n=\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)^n=\cos\frac{4n\pi}{3}+i\sin\frac{4n\pi}{3}$$

$$\begin{aligned}
z_1^n+z_2^n&=\cos\frac{2n\pi}{3}+i\sin\frac{2n\pi}{3}+\cos\frac{4n\pi}{3}+i\sin\frac{4n\pi}{3}\\
&=\left(\cos\frac{2n\pi}{3}+\cos\frac{4n\pi}{3}\right)+\left(\sin\frac{2n\pi}{3}+\sin\frac{4n\pi}{3}\right)i\\
&=2\cos\frac{\frac{2n\pi}{3}+\frac{4n\pi}{3}}{2}\cos\frac{\frac{2n\pi}{3}-\frac{4n\pi}{3}}{2}+2\sin\frac{\frac{2n\pi}{3}+\frac{4n\pi}{3}}{2}\cos\frac{\frac{2n\pi}{3}-\frac{4n\pi}{3}}{2}i\\
&=2\underbrace{\cos n\pi}_{=(-1)^n}\cos\left(-\frac{n\pi}{3}\right)+2\underbrace{\sin n\pi}_{=0}\cos\left(-\frac{n\pi}{3}\right)i\\
&=(-1)^n\cdot 2\cos\frac{n\pi}{3}.
\end{aligned}$$